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Workshop on Wind Energy Computational Analyses

Introduction to Wake Models and Wind Farm Analyses

Ali Ata ADAM, Nilay SEZER-UZOL
Dept. of Aerospace Engineering, METU

21 September 2022

Content:

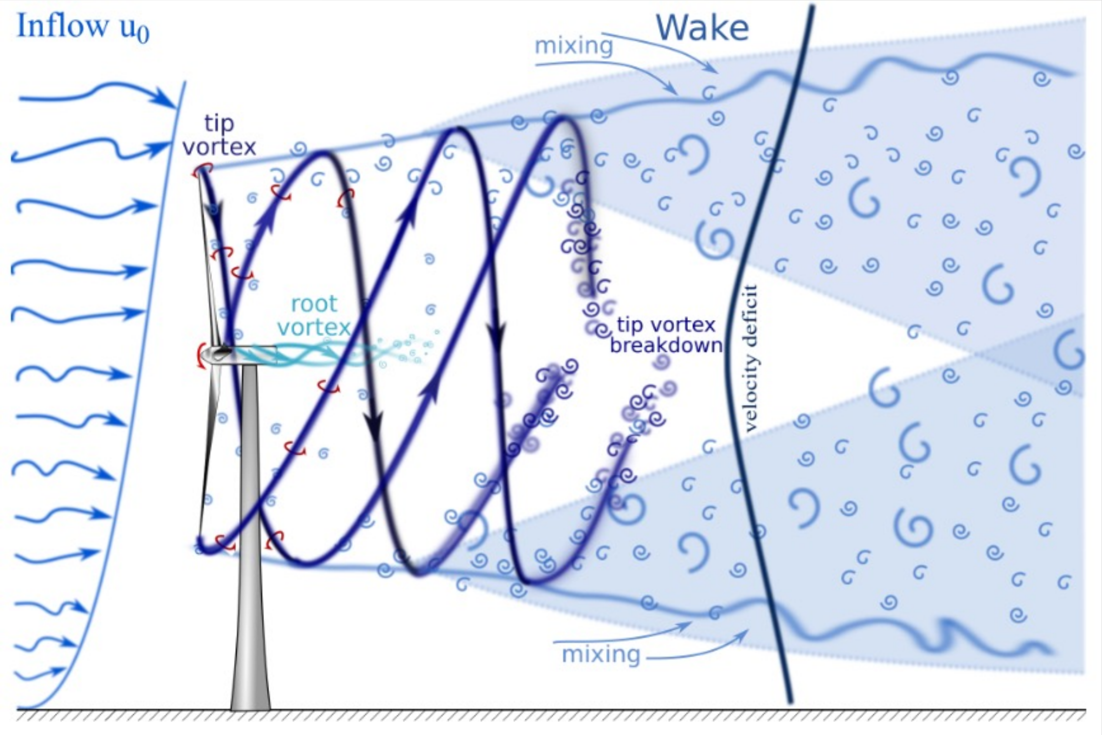
1. Wind Farm Aerodynamics
2. Wake Models
3. Wind Farm Analyses

Content

Wind Farm Aerodynamics

Wakes

- How does wind turbine wakes occur?
    - Wind turbines extract energy from the air
        - Reduced kinetic energy of air
    - Rotor inducing vortices to the flow
        - Increased turbulence
        - Enhanced mixing (turbulent diffusion)



Sketch of the wake downstream a turbine[1]

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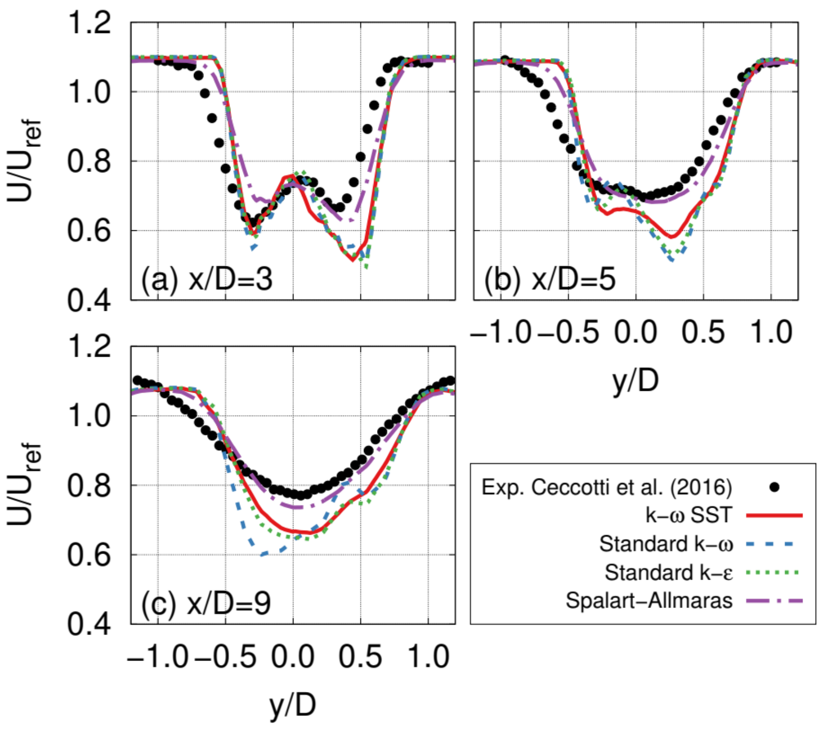
[1] Ingrid Neunaber. “Stochastic investigation of the evolution of small-scale turbulence in the wake of a wind turbine exposed to different inflow conditions in Oldenburg”. Doctorate thesis, Carl von Ossietzky Universität Oldenburg, Jan. 2019.

Wind Farm Aerodynamics

Wakes

- Near the turbine (x/D<3):
  - Unsymmetrical velocity profile due to interactions with non-rotating elements (tower, ground, etc.)
  - Greater maximum and mean velocity deficit
  - Effects of flow interactions with turbine components are present.
- Away from the turbine (x/D>6):
  - Greater wake diameter due to mixing
  - Symmetrical Gaussian velocity profile
  - Less maximum velocity deficit and mean velocity deficit than near the turbine
  - Mean velocity lower than the freestream
  - Greater turbulence level than the freestream



Wake velocity profiles at the hub height for a single-turbine configuration of NTNU model turbine at x/D=3 (a), x/D=5 (b), and x/D=9 (c) downstream of the turbine[2]

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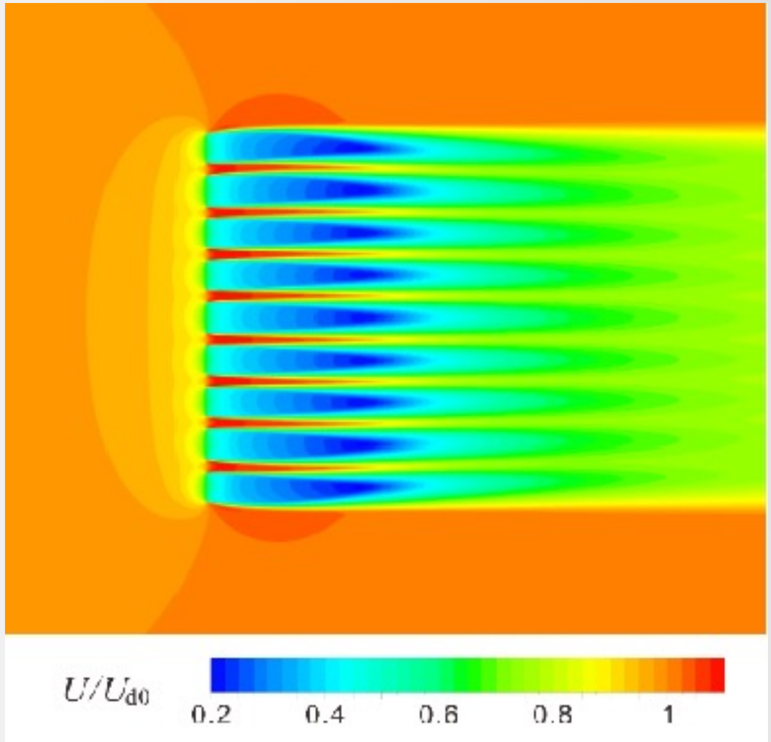
[2] Ali Ata Adam, et al. “Determination of Wake Shapes Behind a Wind Turbine through CFD Simulations and Comparison with Analytical Wake Models.” 11th Ankara International Aerospace Conference (Sep. 2021)

Wind Farm Aerodynamics

Wind Farm Interactions

- The wind farm effects can be classified into[3]:
    - Wake effects (-)
    - Local blockage effects (+)
    - Farm blockage effects (farm-scale flow induction) (-)



Streamwise velocity contours around 9 actuator discs[4]

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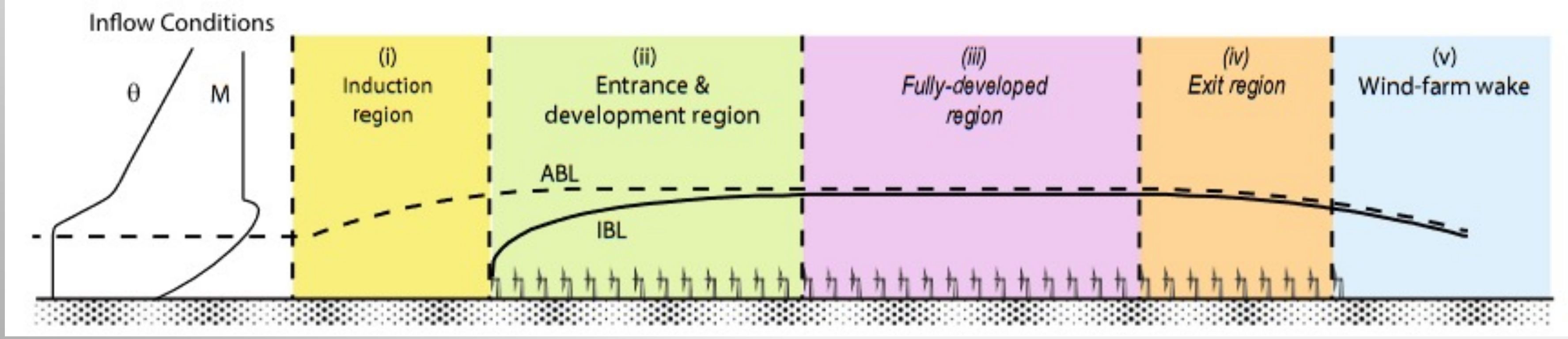
[3] Takafumi Nishino. “Wind farm aerodynamics”

[4] Takafumi Nishino and Scott Draper. “Local blockage effect for wind turbines”. In: *Journal of Physics: Conference Series* 625 (2015), pp. 012010

Wind Farm Aerodynamics

Wind Farm Interactions

- Flow regions inside a wind farm[4]:
  - Wind-farm induction zone: blockage effect induced by the wind farm producing a deceleration of the incoming boundary-layer flow
  - Entrance and flow development region: formation of turbine-wake flows and internal boundary layer (IBL)
  - Fully-developed region: BL flows adjusted to the wind farm
  - Exit region: large accelerating exit region
  - Wind-farm wake region



Different flow regions caused by the interaction of a very large wind farm with a conventionally-neutral ABL[5]

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[5] Fernando Porté-Agel, Majid Bastankhah and Sina Shamsoddin. "Wind-Turbine and Wind-Farm Flows: A Review". In: *Boundary-Layer Meteorology* 174 (2020), pp 1-59. doi: 10.1007/s10546-019-00473-0

Wind Farm Aerodynamics

Wind Farm Interactions

- Wind farm array efficiency depends on many variables:
  - Wind characteristics
  - Turbine rotor design
  - Rotor/rotor, rotor/flow, non-rotor components/flow interactions
  - Wind farm control (wake mitigation)
  - Wind farm layout
  - Terrain and surface conditions
  - Wind farm size (development of IBL)

Horns Rev wind farm. Photographer: Christian Steiness

Introduction to Wake Models and Wind Farm Analyses

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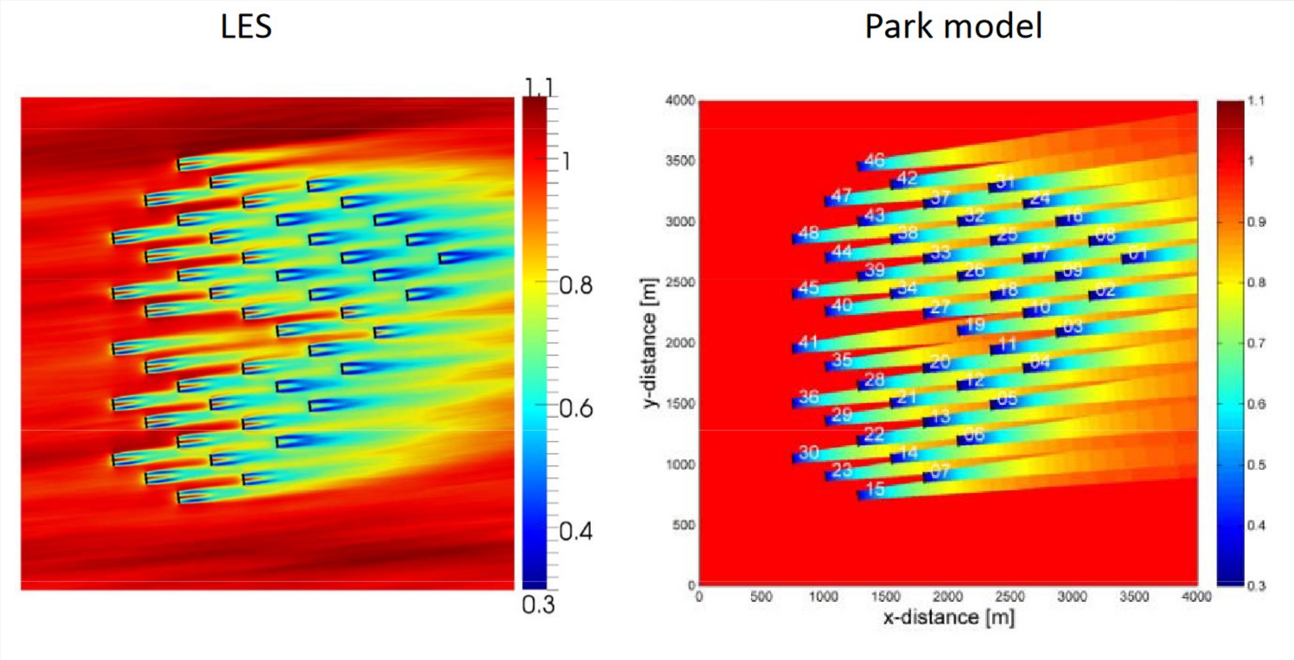
    1. Wakes
    2. Wind Farm Interactions
2. Wake Models
    1. Theory
    2. Wake Superposition Methods
3. Wind Farm Analyses

Wake Models

- Wind farms consist many wind turbines
  - Through meshing
  - Fine resolution required due to large turbulence scales
  - Complex interactions requiring high-fidelity models
  - **High computational cost**
    - May not be feasible for wind farm control or array optimization works
- As a computationally cheap approach: engineering wake models



Normalized velocity contours at the hub height for the LES actuator line (left) and engineering wake model (right) for the Lillgrund wind farm[6]

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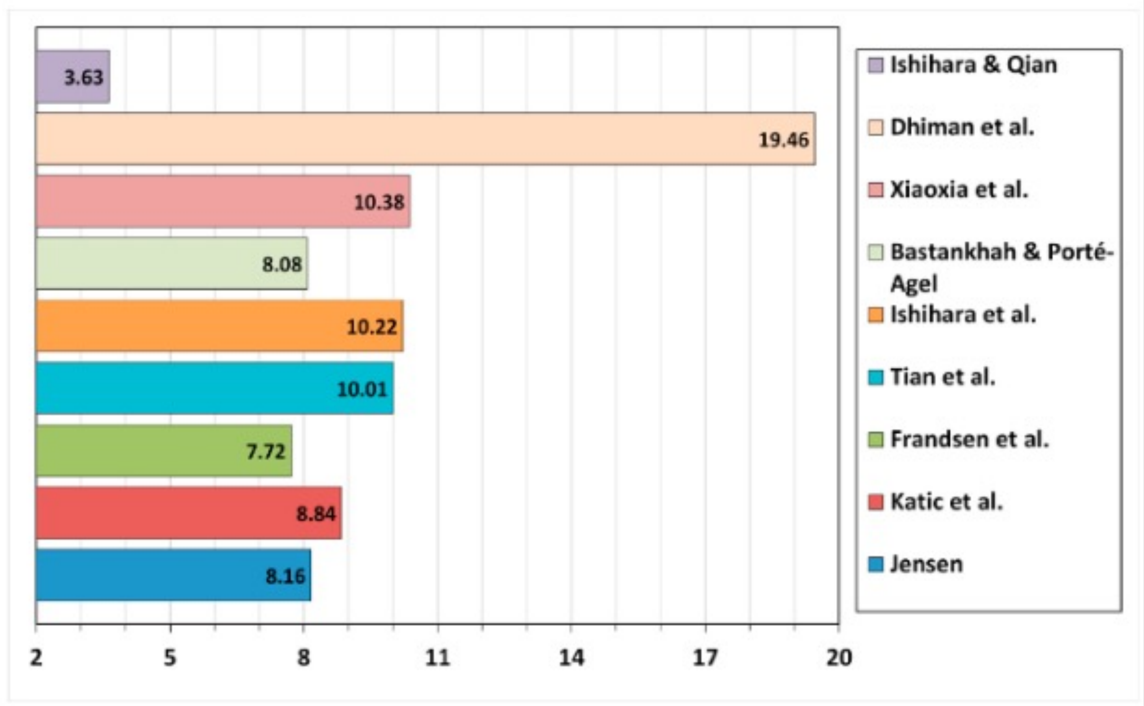
[6] Craig Smith, et al. “Complex wake merging phenomena in large offshore wind farms”. In: *AMS 20th Symposium on Boundary Layers and Turbulence* (2012), Boston, USA.

Wake Models

- As a computationally cheap approach: engineering wake models
  - Velocity profile at the downstream of a turbine is computed with analytical equations and empirical relations.
  - Semi-analytical superposition models are applied to get a wind farm flow.



Average velocity deviation of several wake models for two experimental cases and one LES case[7]

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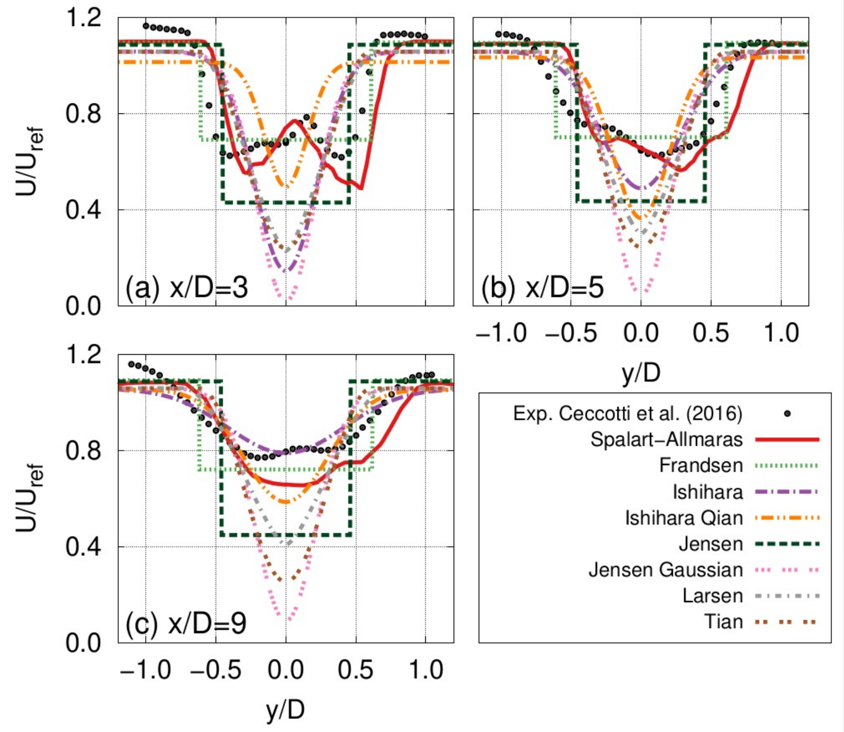
[7] John K. Kaldellis, Panagiotis Triantafyllou and Panagiotis Stinis. “Critical evaluation of Wind Turbines’ analytical wake models”. In: *Renewable and Sustainable Energy Reviews* 114 (Jul. 2021), pp. 110991. doi: 10.1016/j.rser.2021.110991

Wake Models

Theory

- Most wake models are derived by applying the conservation equations within a control volume with assumptions.
  - Usually, some terms (pressure, gravity, viscous, etc.) in the momentum equations are neglected.
  - Empirical relations (wake expansion coeff., wake radius, etc.) are introduced.
    - Extracted from experimental measurements
  - Mostly, the turbine attributes (thrust coefficient, hub height) and atmospheric conditions (turbulence intensity) are taken into account.
  - Earlier methods proposed a top-hat velocity profile, while the modern ones do a Gaussian profile.



Wake velocity profiles from several wake models at the hub height for a single-turbine configuration of NTNU model turbine at x/D=3 (a), x/D=5 (b), and x/D=9 (c) downstream of the turbine[8]

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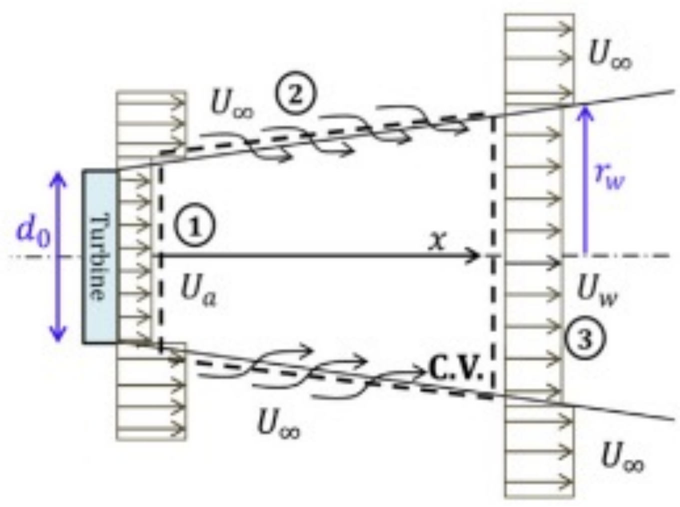
[8] Ali Ata Adam et al., op. cit.

Wake Models

Theory



Derivation of the Bastankhah and Porté-Agel wake model[9]:

- Apply conservation of mass and momentum for the wind turbine wake with neglecting the viscous and pressure terms in the momentum equation:

$$\rho \int u_W (u_\infty - u_W) dA = T = \frac{1}{2} C_T \rho A_0 u_\infty^2$$

- Assume a Gaussian shape for the velocity deficit:

$$\frac{\Delta u(x,r)}{u_\infty} = 1 - \frac{u_W(x,r)}{u_\infty} = C(x) e^{-\frac{r^2}{2\sigma^2}}$$

$C(x)$ represents the maximum normalized velocity deficit

- Integration from 0 to ∞:

$$8\left(\frac{\sigma}{d_0}\right)^2 C(x)^2 - 16\left(\frac{\sigma}{d_0}\right) C(x) + C_T = 0$$

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[9] Majid Bastankhah and Fernando Porté-Agel. "A new analytical model for wind-turbine wakes". In: *Renewable Energy* 70 (Oct. 2014), pp. 116-123. doi:10.1016/j.renene.2014.01.002

Wake Models

Theory

Derivation of the Bastankhah and Porté-Agel wake model[9]:

- Solving the last equation for C(x):

$$C(x) = 1 - \sqrt{1 - \frac{C_T}{8(\sigma/d_0)^2}}$$

- Assume a linear expansion for the wake region:

$$\frac{\sigma}{d_0} = k^* \frac{x}{d_0} + \epsilon$$

- Thus, the velocity deficit becomes equal to:

$$\frac{\Delta u(x,y,z)}{u_\infty} = \left(1 - \sqrt{1 - \frac{C_T}{8\left(\frac{k^* x}{d_0} + \epsilon\right)^2}}\right) \exp\left(-\frac{1}{2\left(\frac{k^* x}{d_0} + \epsilon\right)^2}\left[\left(\frac{z - z_h}{d_0}\right)^2 + \left(\frac{y}{d_0}\right)^2\right]\right)$$

$k^* = \frac{\partial \sigma}{\partial x}$ is the wake growth rate (another work relates this term with the ambient turbulence intensity[10])

$\epsilon = f(C_T)$ is the normalized initial wake half-width (derived from the Frandsen wake model and fit from a LES dataset)

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[9] Majid Bastankhah and Fernando Porté-Agel. "A new analytical model for wind-turbine wakes". In: *Renewable Energy* 70 (Oct. 2014), pp. 116-123. doi:10.1016/j.renene.2014.01.002

[10] A. Niayifar and F. Porté-Agel. ”Analytical Modeling of Wind Farms: A New Approach for Power Prediction”. In: *Energies* 9 (Sep. 2016)

Wake Models

Wake Superposition Models

- Wake velocity distribution for the multiple turbines is determined by,
  - calculating the individual turbine wakes separately, as if they are isolated
  - then for each discretized point, combining the wakes passing that point with a superposition method.
- Some of the commonly utilized superposition methods:

1. Linear sum (Global): $\frac{u(x,y,z)}{u_{\mathrm{ref}}} = 1 - \sum_{i=1}^{n}\left(1 - \frac{u_i(x,y,z)}{u_{ref}}\right)$
2. Linear sum (Local): $\frac{u(x,y,z)}{u_{\mathrm{ref}}} = 1 - \sum_{i=1}^{n}\left(\frac{u_{iw}}{u_{ref}} - \frac{u_i(x,y,z)}{u_{ref}}\right)$
3. Root sum square (Global): $\frac{u(x,y,z)}{u_{\mathrm{ref}}} = 1 - \sqrt{\sum_{i=1}^{n}\left(1 - \frac{u_i(x,y,z)}{u_{ref}}\right)^2}$
4. Root sum square (Local): $\frac{u(x,y,z)}{u_{\mathrm{ref}}} = 1 - \sqrt{\sum_{i=1}^{n}\left(\frac{u_{iw}}{u_{ref}} - \frac{u_i(x,y,z)}{u_{ref}}\right)^2}$

u_{iw} is the local average velocity within the rotor swept area incoming to the turbine i

Wake Models

Wake Superposition Models

- Not only the incoming velocity but also the turbulence level upstream of the second turbine is altered which may affect some parameters within the model.
  - Some models calculate the wake turbulence level within the wake, some do not.
  - For turbines affected by a wake, the ambient turbulence level can be replaced with the wake turbulence level at the location where the turbine is located.

- The wake superposition models lack a theoretical basis.
  - While the linear models preserve mass continuity, the root sum square models take the energy conservation as their goal.
  - The shown models are empirically derived.
  - There are additional models derived with analytical, numerical, empirical, or machine-learning bases[11]

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[11] Maulidi Barasa et al. "The balance effects of momentum deficit and thrust in cumulative wake models". In: *Energy* 246 (2022), pp. 123399. doi:10.1016/j.energy.2022.123399

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Wind Farm Analyses

- Models for wind farm analyses
  1. Wake models
  2. Blade Element Momentum Theory
  3. Lagrangian-based models
  4. Potential flow-based models
  5. Actuator disk/line/surface
  6. Direct modeling

Greater accuracy
More expensive

1. Wake models:
   - C_T and u_∞ data => Wake velocity profile => C_P of each turbine => Wind farm power generation
   - Simplest and cheapest approach
   - Complex wake structures and local blockage effect cannot be predicted.
   - Useful for optimization, wind farm layout, and wind farm control applications.

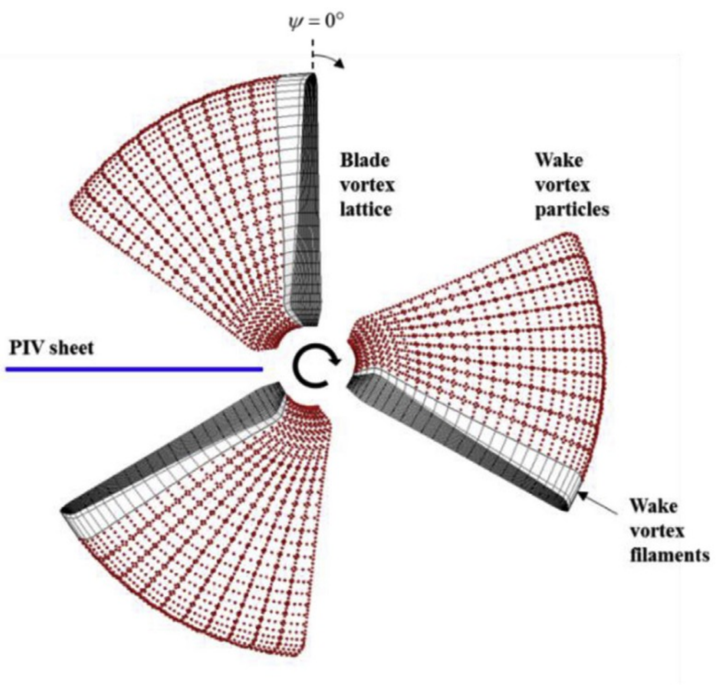
Wind Farm Analyses

2. Blade Element Momentum Theory:

- Combines the actuator theory with 2D blade elements
- The effects of tip losses and the wake rotation can be included.
- Requires airfoil performance data
    - Difficult to get a 2D dataset with rotational and 3D effects

3. Lagrangian-based models

- The vortex-lattice and vortex-particle methods
- Wake is described with sheets or particles
- Blade is represented with lifting lines or surface
- Inviscid and incompressible flow assumptions



Rotor blade and wake modeling in the vortex-lattice method[12]

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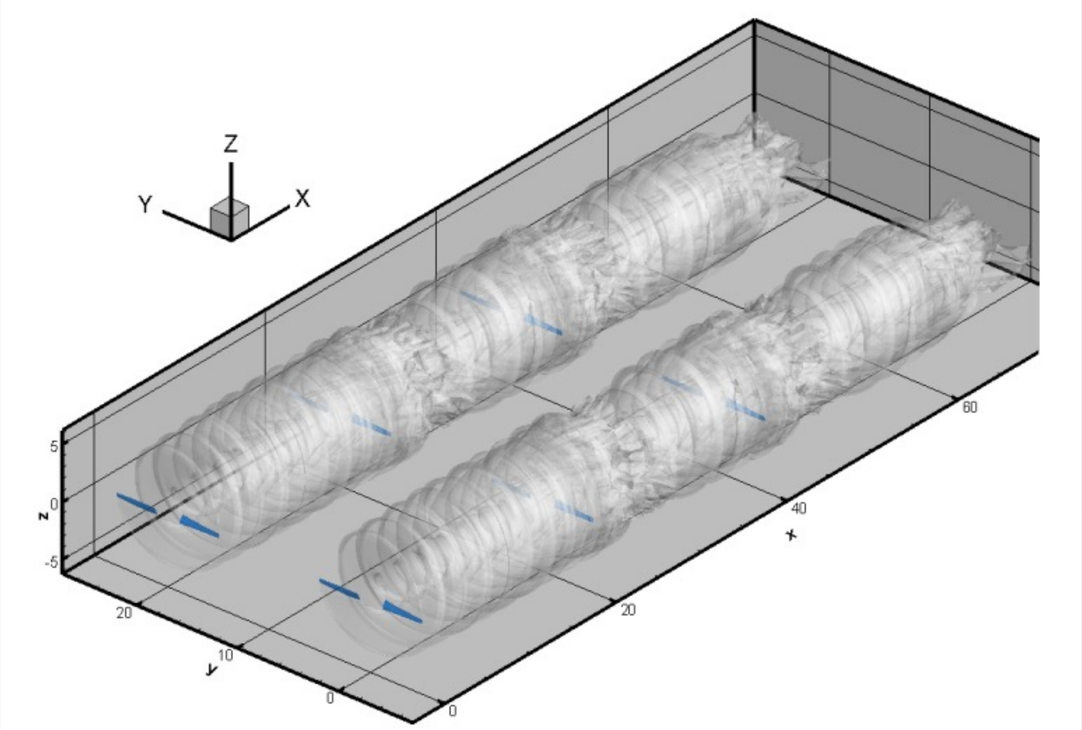
[12] Hakjin Lee and Duck-Joo Lee. “Numerical investigation of the aerodynamics and wake structures of horizontal axis wind turbines by using nonlinear vortex lattice method”. In: *Renewable Energy* 132 (2019), pp. 1121-1133. doi:10.1016/j.renene.2018.08.087

Wind Farm Analyses

4. Potential flow-based models
    - Panel method
    - Viscous effects can be included with a B/L code and a free-wake
    - Blade geometry is considered.
    - Only surface discretization
    - Incompressible flow assumption
    - Uncertainty in vortex core modeling and expensive vortex filament interaction calculations



Multiple turbine simulation of 6 NREL Phase VI rotors with a panel code[13]

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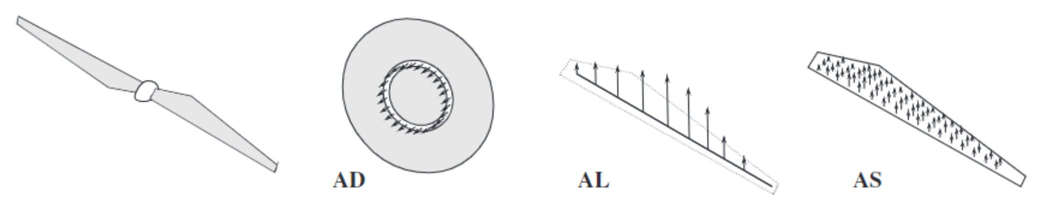
[13] M. Türkal et al.. “GPU Based Fast Free-Wake Calculations For Multiple Horizontal Axis Wind Turbine Rotors”. In: *Journal of Physics: Conference Series* 524 (2014), pp. 012100. doi:10.1088/1742-6596/524/1/012100

Wind Farm Analyses

5. Actuator disk/line/surface
   - Blade is represented with an actuator force source
     - No need to discretize the blade
     - Less pre-processing time required
   - Volume mesh is needed for RANS/LES computation
   - Requires an accurate actuator model and blade data



Illustriation of the full-resolved blade (first), actuator disk (second), line (third), and surface (fourth) concepts[14]

6. Direct modeling
   - The rotor components are modelled as they are.
   - Requires high-quality mesh with a fine boundary-layer representation
   - Extremely costly (both pre-processing and flow computation) in an optimization or similar applications

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[14] B. Sanderse, S.P. van der Pijl and B. Koren. “Review of computational fluid dynamics for wind turbine wake aerodynamics”. In: *Wind Energy* 14 (2011), pp. 799-819. doi:10.1002/we.458

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This project has received funding from the European High-Performance Computing Joint Undertaking (JU) under grant agreement No 951732. The JU receives support from the European Union’s Horizon 2020 research and innovation programme and Germany, Bulgaria, Austria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, Greece, Hungary, Ireland, Italy, Lithuania, Latvia, Poland, Portugal, Romania, Slovenia, Spain, Sweden, United Kingdom, France, Netherlands, Belgium, Luxembourg, Slovakia, Norway, Switzerland, Turkey, Republic of North Macedonia, Iceland, Montenegro