

DIALOGUES IN THE MIDST

Workshop in the MIDST of

**Simulating soft matter
with dissipative particle dynamics**

Gökhan Kaçar
Trakya University



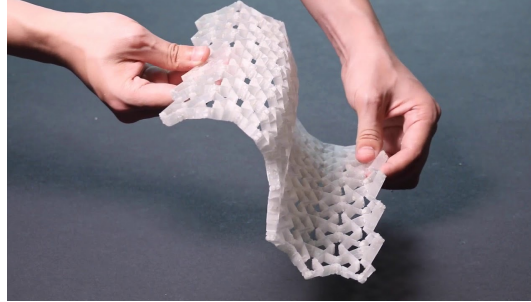
Sabancı
Universitesi

Outline

- Soft matter
 - Definition and examples
- Molecular dynamics
 - Definitions and some basic theory
- Part 1
 - Coarse-graining and coarse-grained simulations: General concepts
- Part 2
 - Dissipative Particle Dynamics method
 - Definition
 - Some examples: Applications on soft matter
- Cases/properties to be studied in the hands-on session

Soft matter

- Materials that can deform?
- Things that do not hurt your hand when you hit them?
- Water and cornstarch mixture?

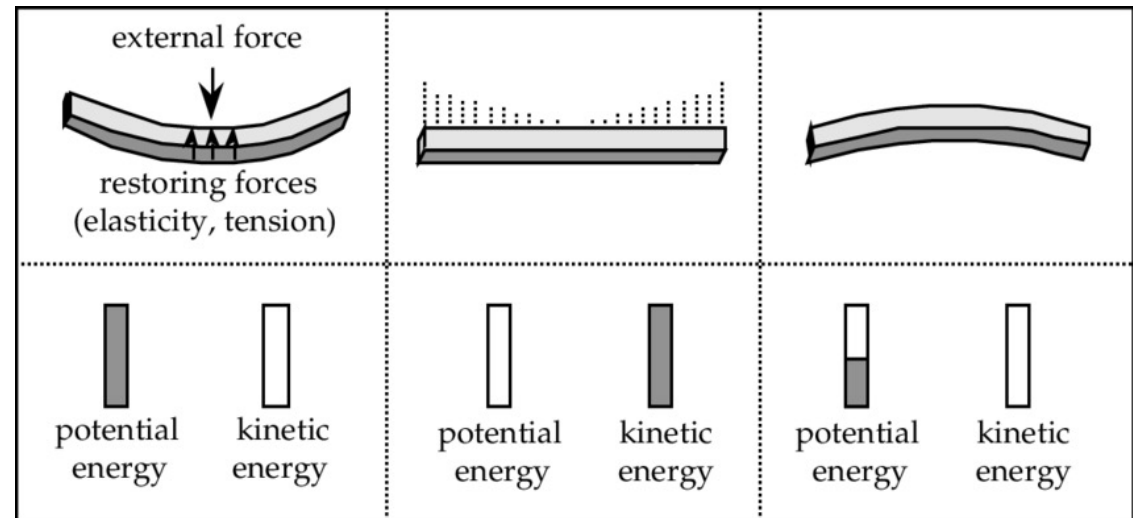
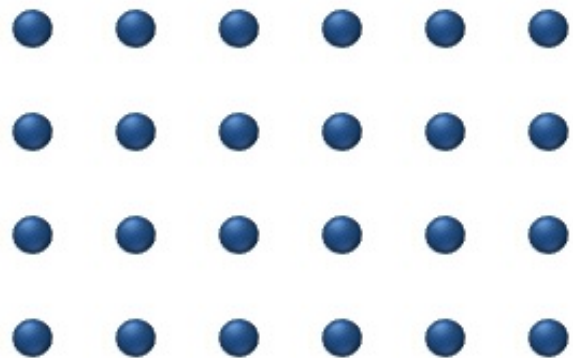


- Fluid or solid?

Soft matter

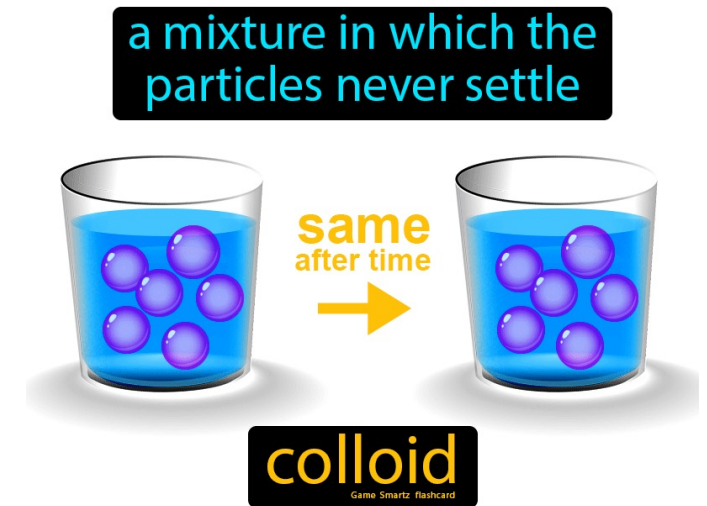
Definition

Materials that are easily deformed by thermal fluctuations and external forces



Soft matter

- hair gel, mayonnaise, shaving cream, colloidal crystals, polymer solutions and blends



Sources:
wikipedia.org
barbasol.com
recipeclinic
pinterest

Soft matter

Similar to ordinary solids and liquids?

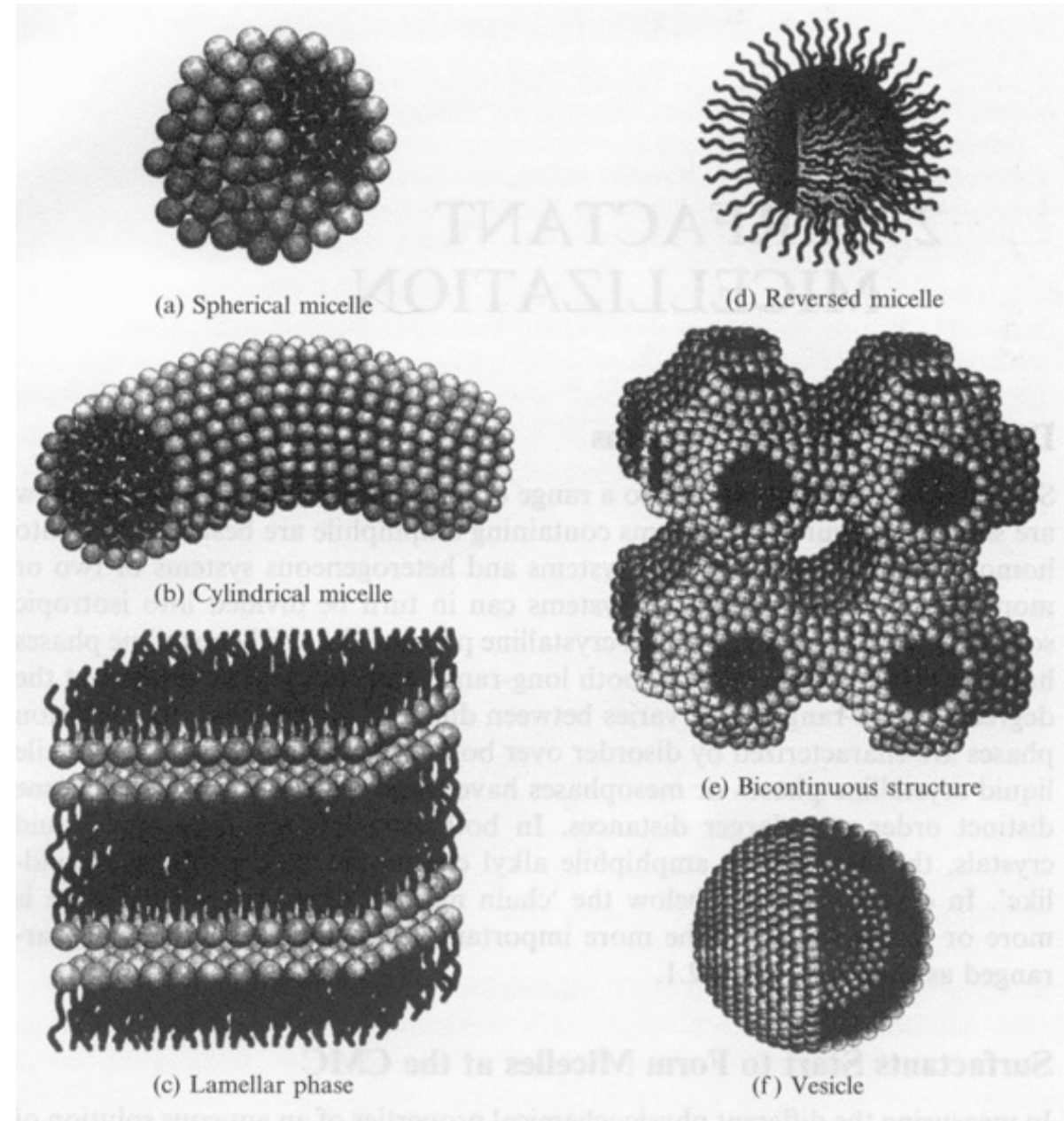
NO!

Individual molecules seem random and disordered 'like a liquid'

Zoom in



Source: Flaticon.com



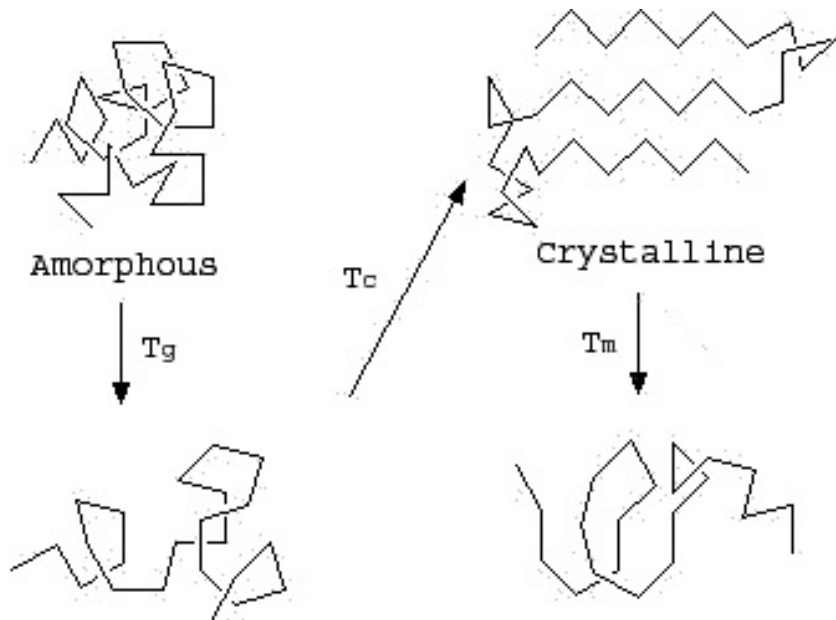
Soft matter

Properties

Large response function

Small change can dramatically change the properties

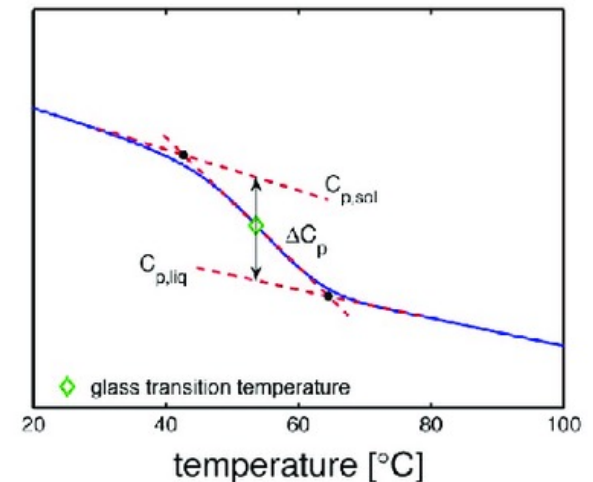
Environmental changes (temperature, light, magnetic or electric field)



“Self-assembly”

T_g marks the temperature at which an amorphous polymer changes from rigid solid to a more flexible rubber

$$T_m > T_c > T_g$$



Soft matter

Length-scales

Soft matter length scales fall in between atomistic and macroscopic scales

Mesosopic length scale

However, fluctuations from Brownian motion are still important!!!

Brownian Motion



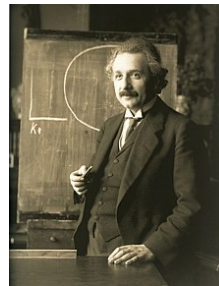
Botanist Robert Brown (1773 - 1858)

Noticed the irregular motion of pollen granules in water

(Roman philosopher Lucretius' famous poem already gives some notice to the motion of dust particles in air)

Albert Einstein and Polish physicist Marian Smoluchowski explained the Brownian motion

Statistical mechanics-based explanation of the movement of pollen by the force exerted from individual water molecules



Soft matter

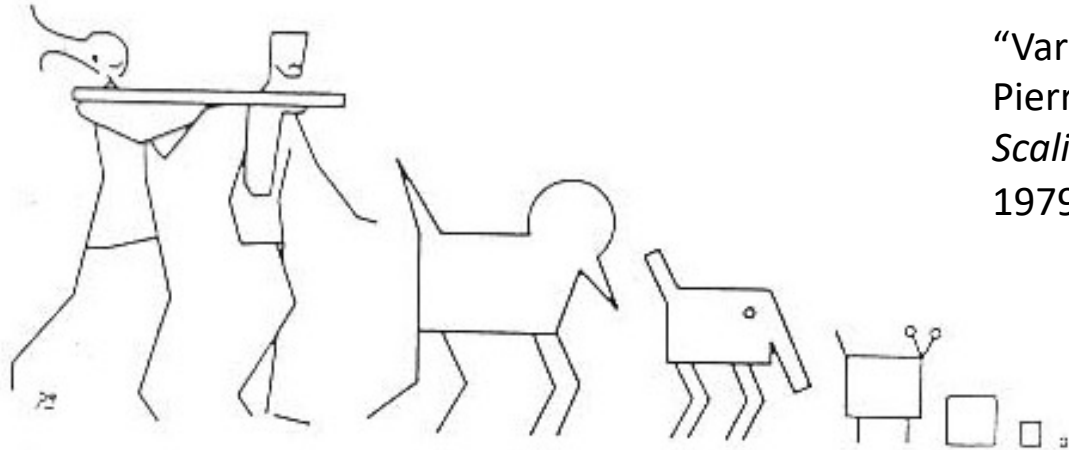
Definition (again) – Complex matter or complex fluids

Molecular building blocks

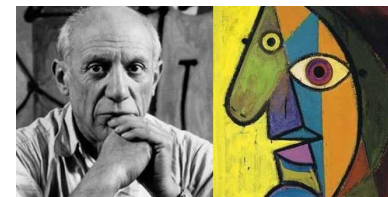
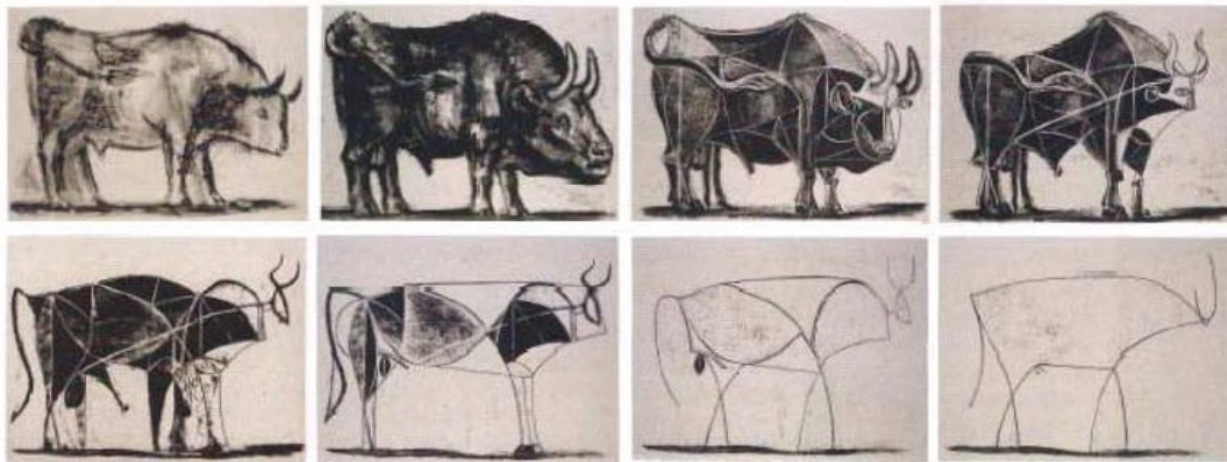
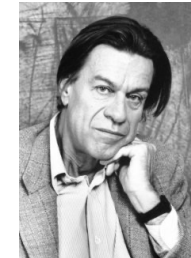


Pierre-Gilles de Gennes

Soft matter



“Various animals attempting to follow a scaling law” by Pierre-Gilles de Gennes (Nobel prize in physics 1991) *Scaling Concepts in Polymer Physics*, Cornell University Press 1979.



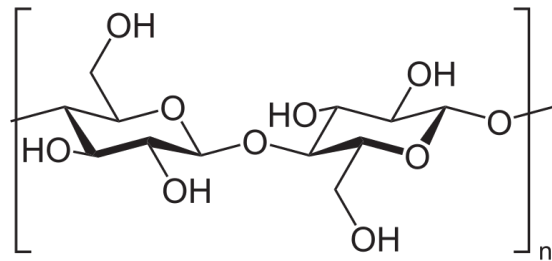
Picasso, *The Bull*, 1945.

“According to Locke (John Locke, 1632-1704), the complex idea of a substance is a collection of simple ideas that is believed capable of existing independently. Observing in experience that several features recur together frequently, we suppose that there must be some common subject that has all of them.”, The Philosophy Pages by Garth Kemerling.

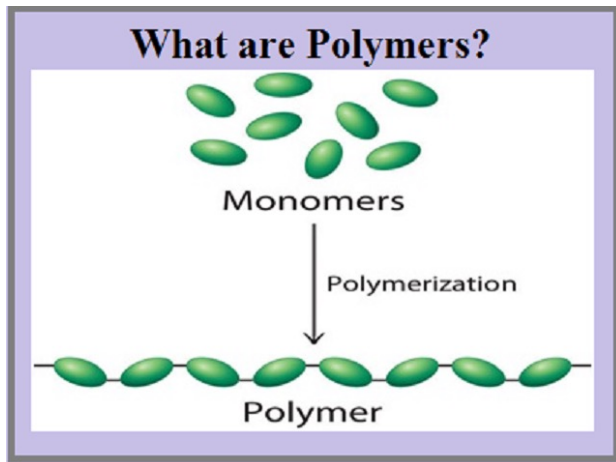
Complex *soft matter*

Why are we interested in soft matter?

Complexity is the hallmark of living systems/natural and synthetic soft materials

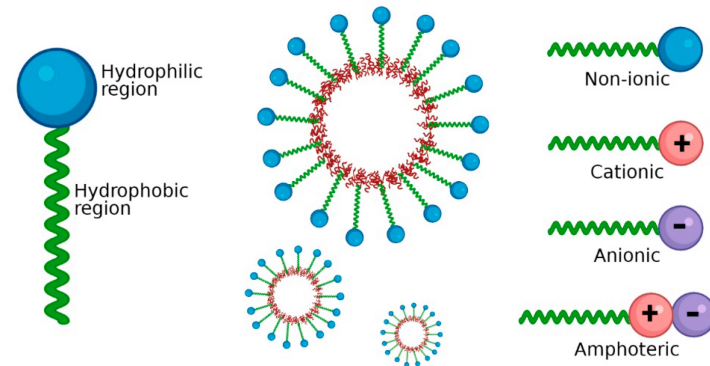


Cellulose



wikipedia.org

a) Surfactant structure b) Nanoparticle coating c) Classification of surfactants

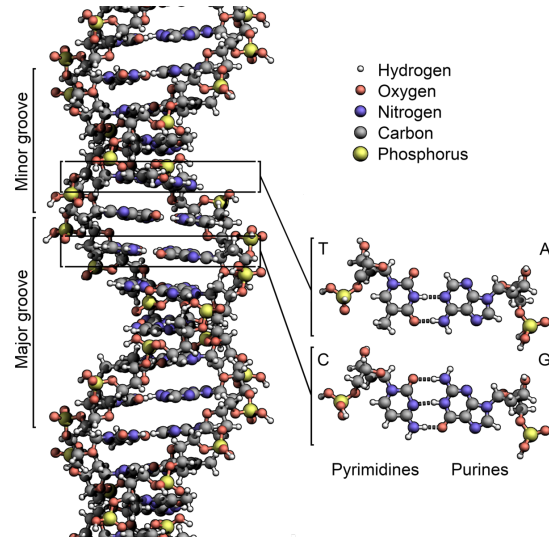


Surfactants

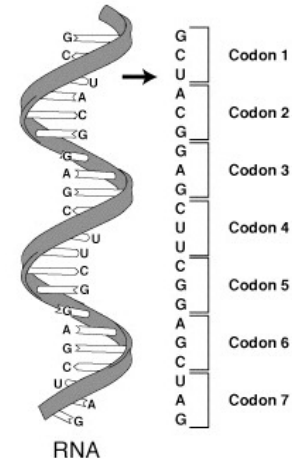
Polymers

Complex *soft matter*

Biological macromolecules

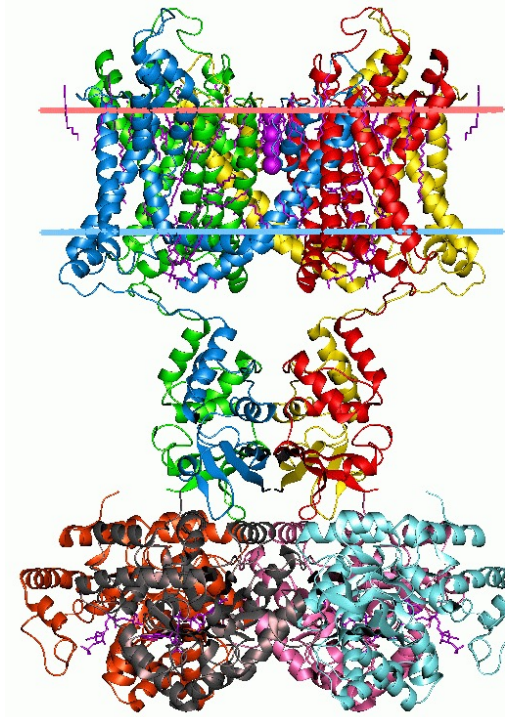


DNA



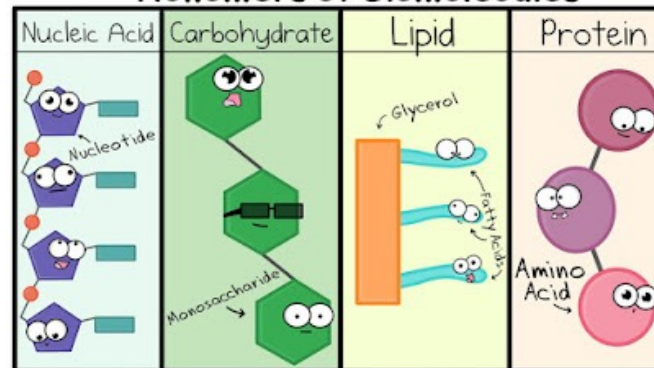
Ribonucleic acid

RNA



V-gated K channel

Monomers of Biomolecules



Amoeba Sisters

#AmoebaGIFs

Complex *soft matter*

Typically composed of nano-/meso- building blocks

Easily deformable when exposed to weak external fields

flow fields (microfluids), mechanical forces, electric/magnetic field, thermal agitation

e.g. paint, blood, milk, spreads and ice cream, ...

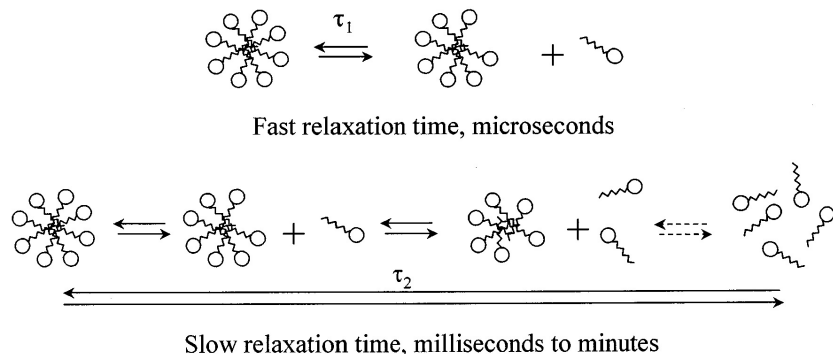
Often formed as a result of self-assembly

Lacking quantitative description of heterogeneous, hierarchical, non-equilibrium soft materials in living systems/synthetic materials

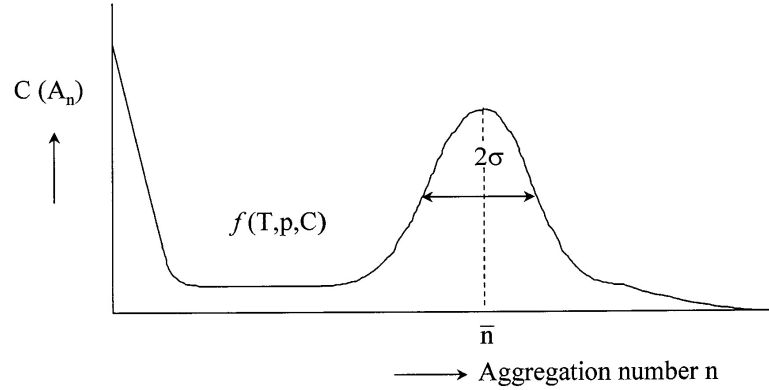
Building blocks → determination of proper ‘scaling laws’

Time and length scale (= static and dynamic properties)

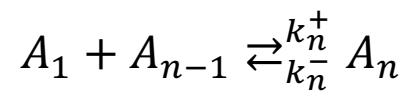
Molecular Self-Assembly



A. Patist et al. / Colloids and Surfaces A: Physicochem. Eng. Aspects 176 (2001) 3-16



Typical size distribution curve of aggregates in a micellar solution.



A: aggregate

Fast relaxation process

$$\frac{1}{\tau_1} = \frac{k^-}{\sigma^2} \left(1 + \frac{\sigma^2}{n} a \right), \quad a = \frac{C - CMC}{CMC}$$

Average micellar lifetime

$$T_m = \tau_2 \frac{na}{1 + \frac{\sigma^2}{n} a} \approx n\tau_2$$

Slow relaxation process

$$\frac{1}{\tau^2} = \frac{n^2}{CMC * R} \left(1 + \frac{\sigma^2}{n} a \right)^{-1}$$

Colloidal Phase Behavior

Many sizes and shapes

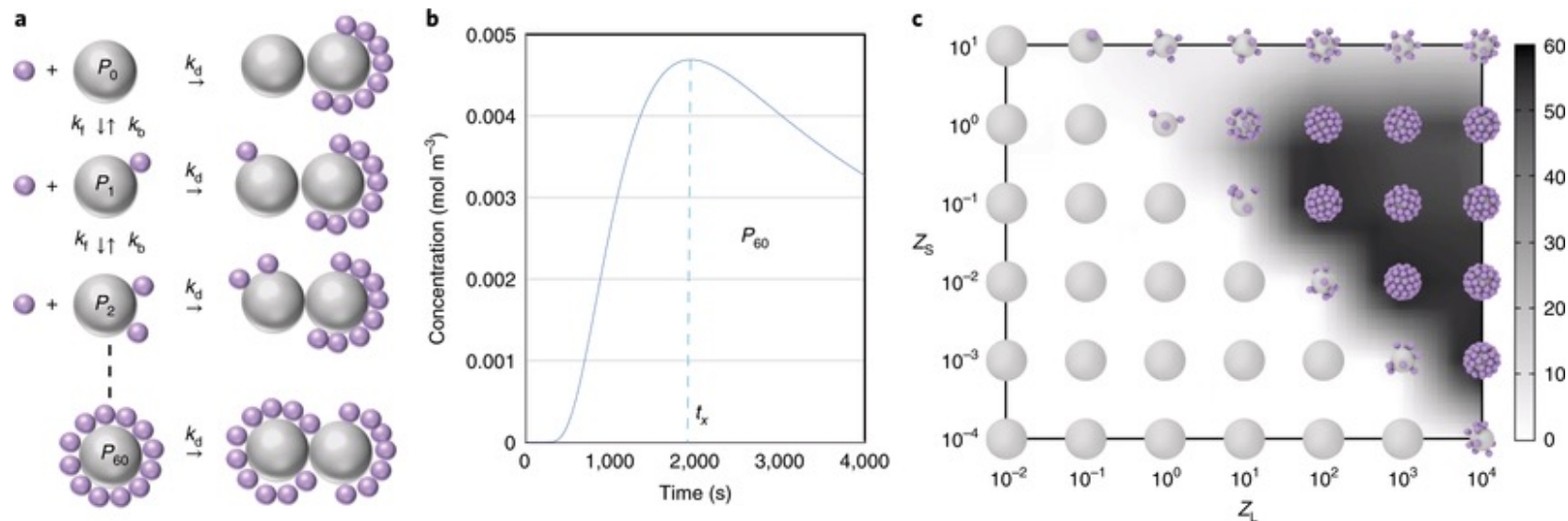
surface charge, pH-salt concentration, polymeric additives, etc.

Self-organization into variety of structures/phases

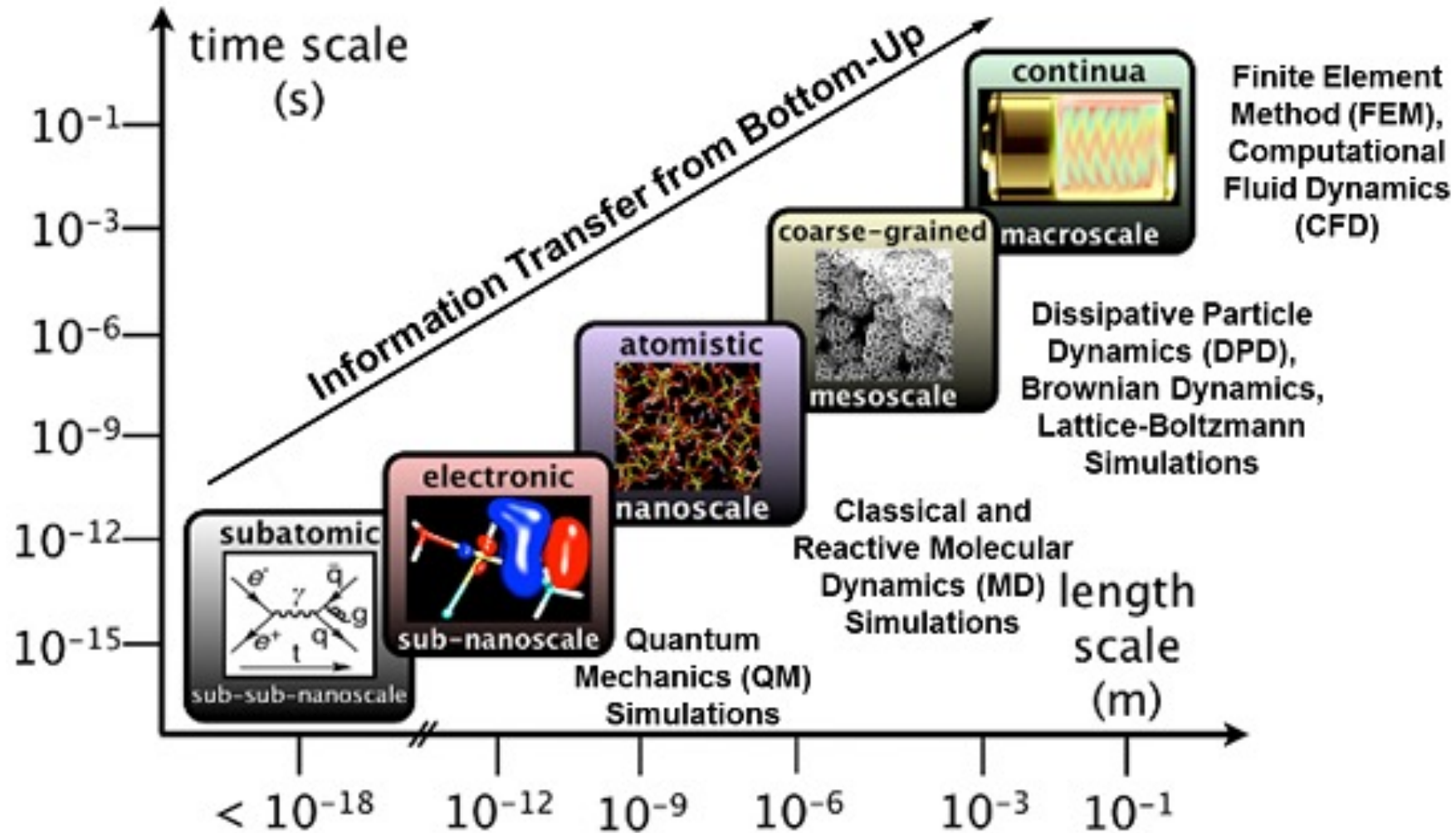
Slower dynamics (even monitored by optical microscope)

Designer building blocks

Molecular-level information to rationalize the experimental design



Molecular-level insight from *molecular simulations*

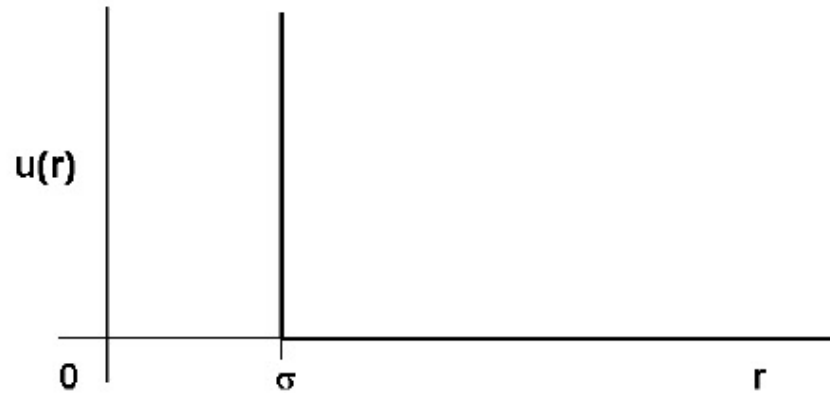


Molecular dynamics

The solution of the classical equations of motion (Newton's equations) for a set of molecules.

$$\mathbf{F}_i = -\nabla_i V_B \quad \vec{F} = m\vec{a}$$

First examples
Hard-sphere system



Berni Alder (1925–2020)
Thomas Wainwright (1927–2007)

IBM 704 at University of California Radiation
Laboratory at Livermore.



Molec

The solut
a set of m

First exam
Hard-sphe

Berni Alder
Thomas W

IBM 704 a
Laborator



ons) for



Molecular dynamics

Method is greatly extended by Aneesur Rahman (1927-1987)

Particles characterized by continuous potentials

Simulations of realistic atomic models



Frank Stillinger (born 1934)

First MD simulation of liquid water



Molecu

Method is gre

Particl

Simula

Frank Stillinge

First N

Eur. Phys. J. H (2022)47:13
<https://doi.org/10.1140/epjh/s13129-022-00043-y>

THE EUROPEAN
PHYSICAL JOURNAL H



Regular Article

The emergence of protein dynamics simulations: how computational statistical mechanics met biochemistry

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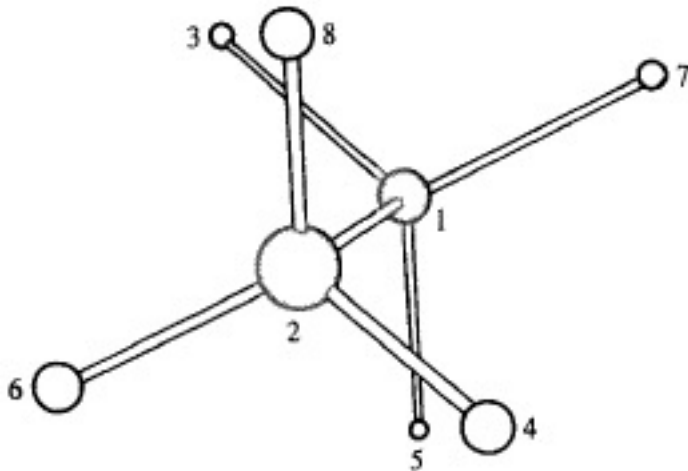
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Abstract In this essay, we aim to illustrate how Martin Karplus and his research group effectively set in motion the engine of molecular dynamics (MD) simulations of biomolecules. This process saw its prodromes between 1969 and the early 1970s with Karplus’ landing in biology, a transition that came to fruition with the treatment of 11-*cis*-retinal photoisomerization and the development of an allosteric model to account for the mechanism of cooperativity in hemoglobin. In 1977, J. Andrew McCammon, Bruce Gelin, and Martin Karplus published an article in *Nature* reporting the MD simulation of bovine pancreatic trypsin inhibitor (BPTI). This publication helped initiate the merger of computational statistical mechanics and biochemistry, a process that Karplus undertook at a later stage and whose beginnings we propose to reconstruct in this article through unpublished accounts of the key people who participated in this endeavor.

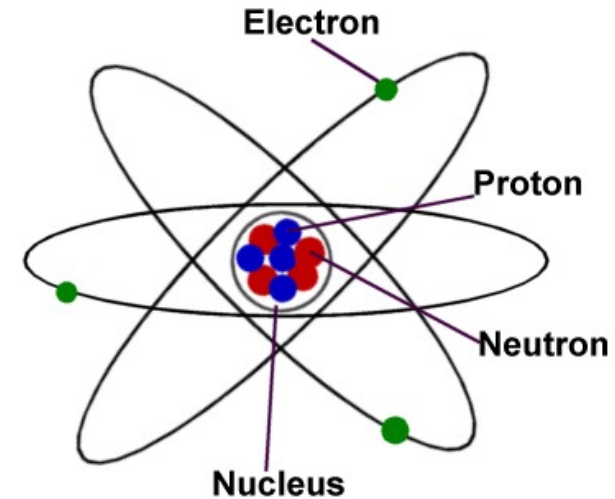


System definition

Cartesian coordinates



C	0.000000	0.000000	0.762209
C	0.000000	0.000000	-0.762209
H	0.000000	1.018957	1.157229
H	-0.882443	-0.509479	1.157229
H	0.882443	-0.509479	1.157229
H	0.000000	-1.018957	-1.157229
H	-0.882443	0.509479	-1.157229
H	0.882443	0.509479	-1.157229



Underlying assumption: Born-Oppenheimer approximation

The wave functions of atomic nuclei and electrons in a molecule can be separated.

The nuclei are much heavier than the electrons. The coordinates of the nuclei in a system are somewhat fixed, while the coordinates of the electrons are dynamic.

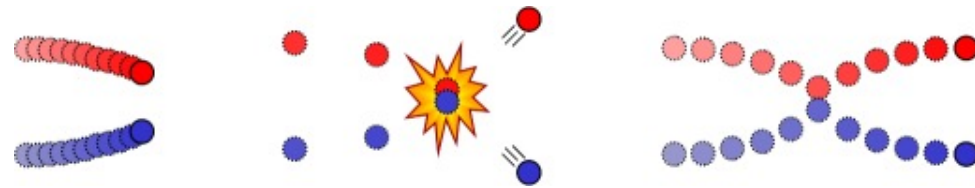
Molecular dynamics

Compute the forces on the particles

Solve the equations of motion

Sample after some timesteps

Simulate collisions



Molecular dynamics

A simple molecular dynamics program

<pre>program md</pre>	simple MD program
<pre>call init</pre>	initialization
<pre>t=0</pre>	
<pre>do while (t.lt.tmax)</pre>	MD loop
<pre> call force(f,en)</pre>	determine the forces
<pre> call integrate(f,en)</pre>	integrate equations of motion
<pre> t=t+delt</pre>	
<pre> call sample</pre>	sample averages
<pre>enddo</pre>	
<pre>stop</pre>	
<pre>end</pre>	

Verlet and v-Verlet algorithms

Equations of motion

$$\mathbf{r}(t + \Delta t) = \mathbf{r}(t) + \mathbf{v}(t)\Delta t + \frac{\Delta t^2}{2m}\mathbf{f}(t) + \frac{\Delta t^3}{3!}\ddot{\mathbf{r}}(t) + \mathcal{O}(\Delta t^4)$$

$$\mathbf{r}(t - \Delta t) = \mathbf{r}(t) - \mathbf{v}(t)\Delta t + \frac{\Delta t^2}{2m}\mathbf{f}(t) - \frac{\Delta t^3}{3!}\ddot{\mathbf{r}}(t) + \mathcal{O}(\Delta t^4)$$

$$\mathbf{r}(t + \Delta t) + \mathbf{r}(t - \Delta t) = 2\mathbf{r}(t) + \frac{\Delta t^2}{m}\mathbf{f}(t) + \mathcal{O}(\Delta t^4)$$

Verlet algorithm

$$\mathbf{r}(t + \Delta t) \approx 2\mathbf{r}(t) - \mathbf{r}(t - \Delta t) + \frac{\Delta t^2}{m}\mathbf{f}(t)$$

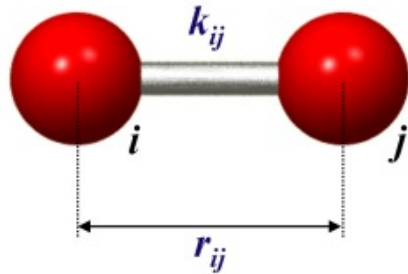
Velocity Verlet algorithm

$$\mathbf{r}(t + \Delta t) \approx \mathbf{r}(t) + \mathbf{v}(t)\Delta t + \frac{\Delta t^2}{2m}\mathbf{f}(t)$$

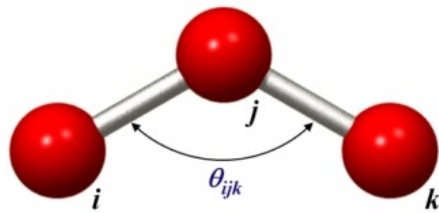
$$\mathbf{v}(t + \Delta t) \approx \mathbf{v}(t) + \frac{\Delta t}{2m}[\mathbf{f}(t + \Delta t) + \mathbf{f}(t)]$$

Modeling molecular-level interactions

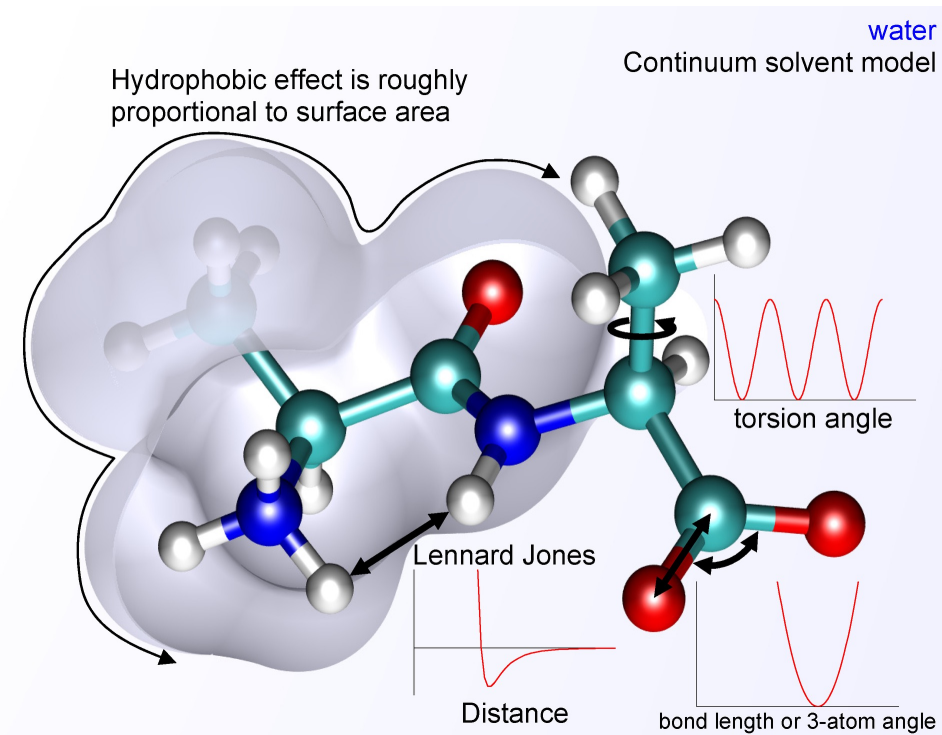
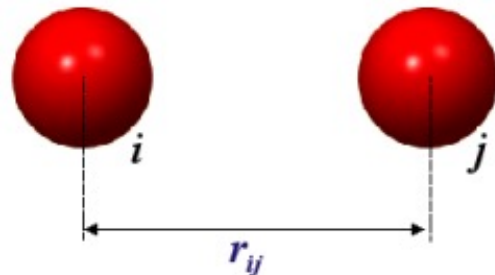
Bonded potential



Angle potential



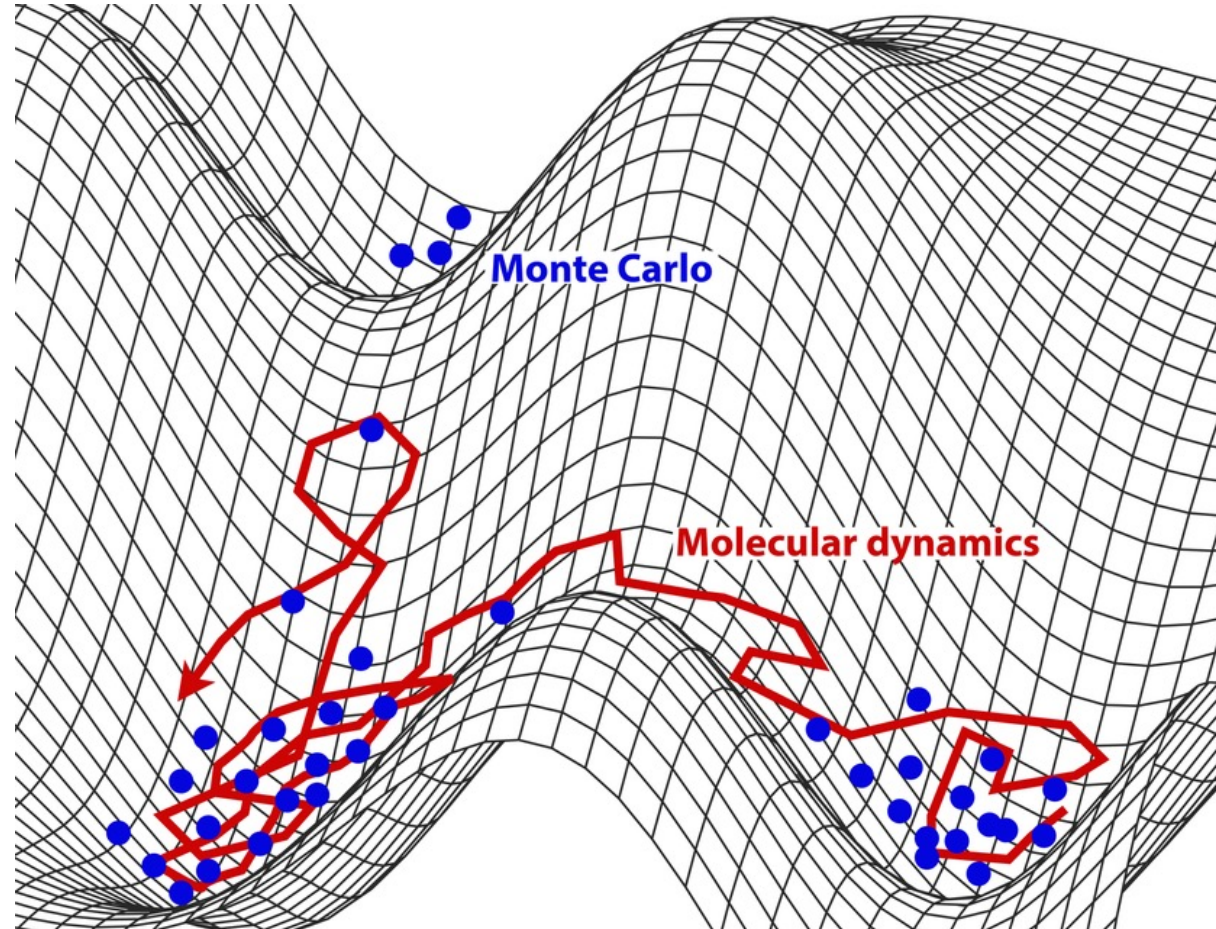
Non-bonded potential



$$V_{vdW} = 4 \epsilon_{ij} \left(\left(\frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}} \right)^6 \right)$$

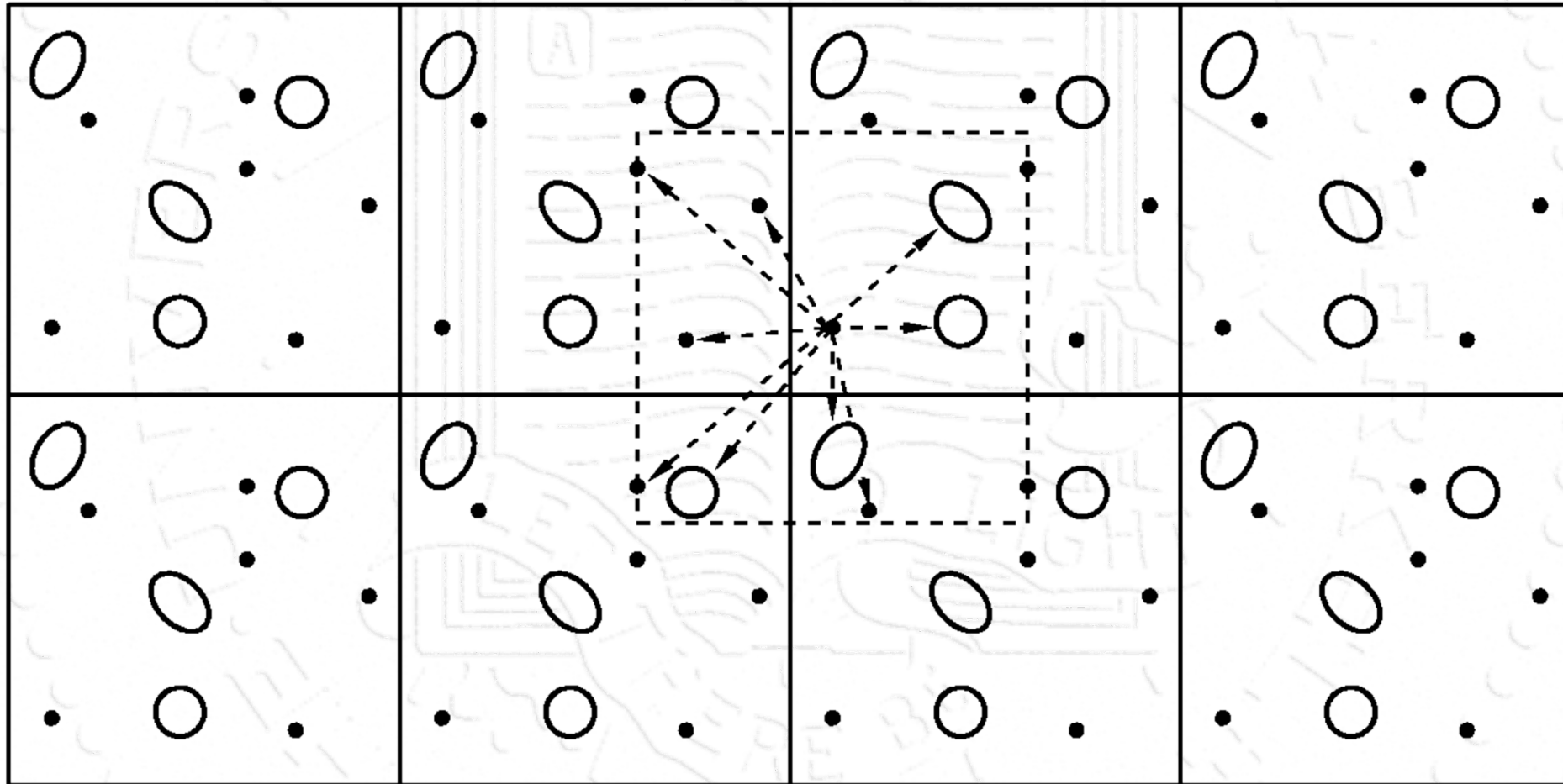
Molecular dynamics

Proper sampling of system's potential energy



Periodic Boundary Conditions

Taking care of close neighbors



Computational efficiency

- The Lennard-Jones potential

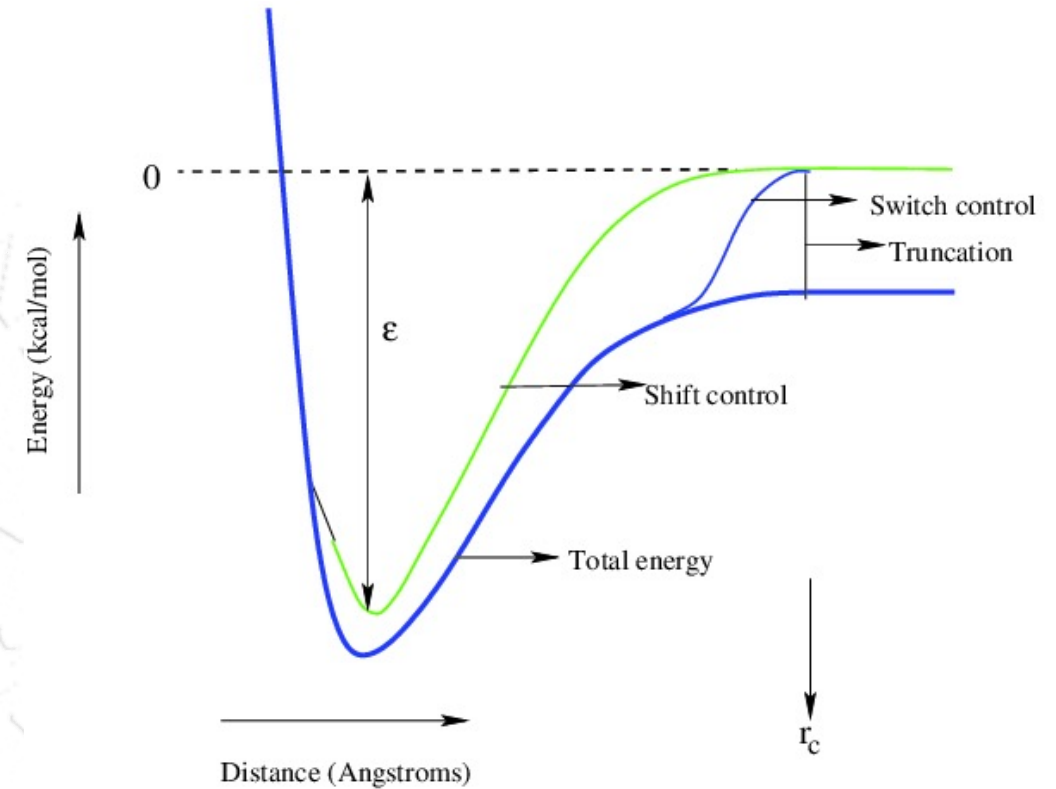
$$u^{LJ}(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right]$$

- The truncated Lennard-Jones potential

$$u(r) = \begin{cases} u^{LJ}(r) & r \leq r_c \\ 0 & r > r_c \end{cases}$$

- The truncated and shifted Lennard-Jones potential

$$u(r) = \begin{cases} u^{LJ}(r) - u^{LJ}(r_c) & r \leq r_c \\ 0 & r > r_c \end{cases}$$



Zoe Cournia, Ph.D. Thesis, 2006.

Minimizing energy

Algorithms to minimize the potential energy of the system

- Steepest Descent
- Conjugate Gradients
- Newton-Raphson

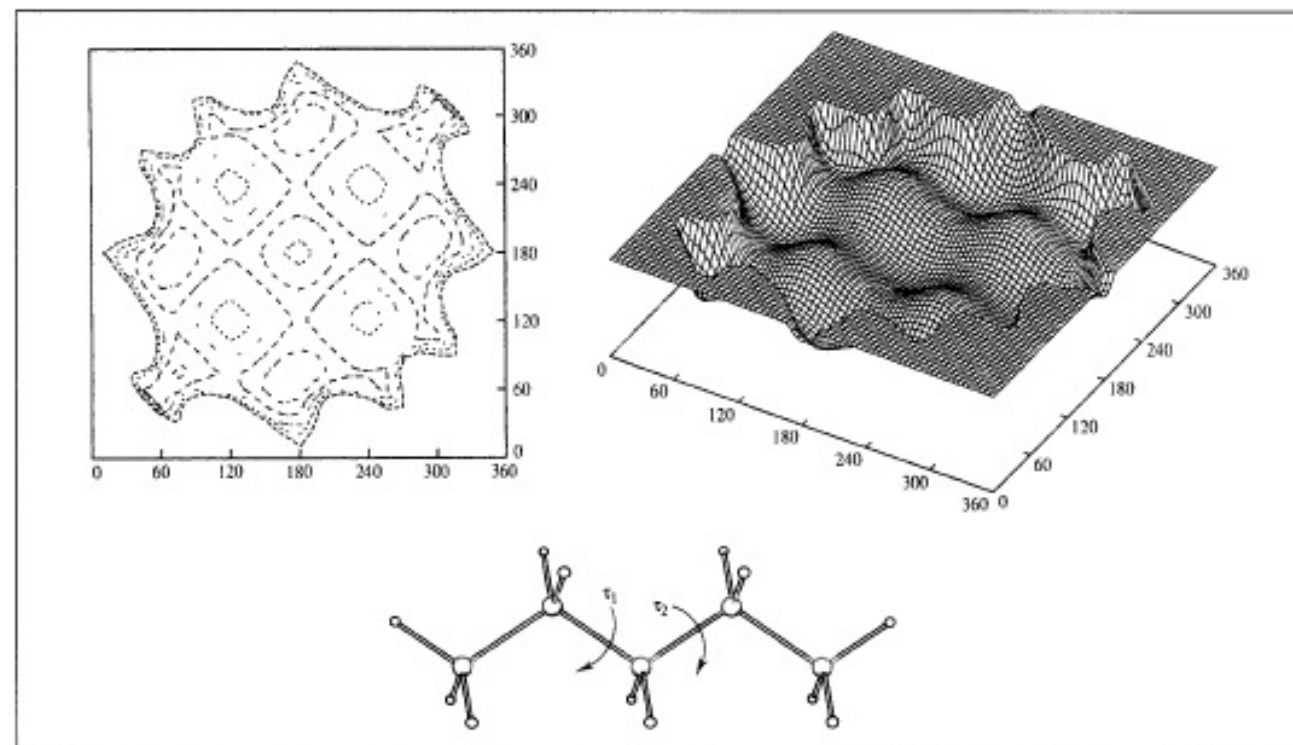
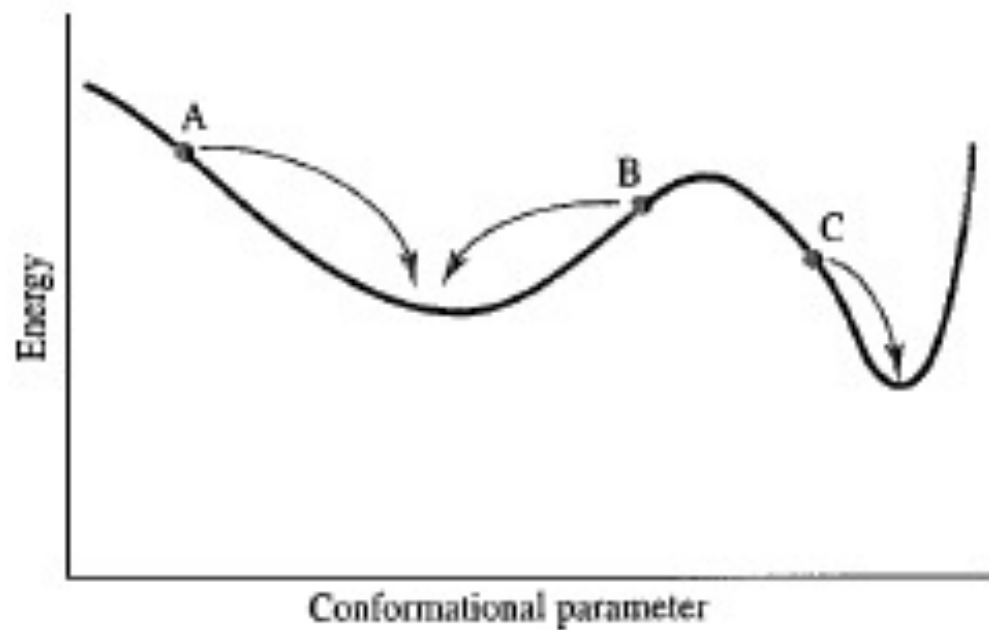


Fig. 5.1 Variation in the energy of pentane with the two torsion angles indicated and represented as a contour diagram and isometric plot. Only the lowest-energy regions are shown.

Which one to use?

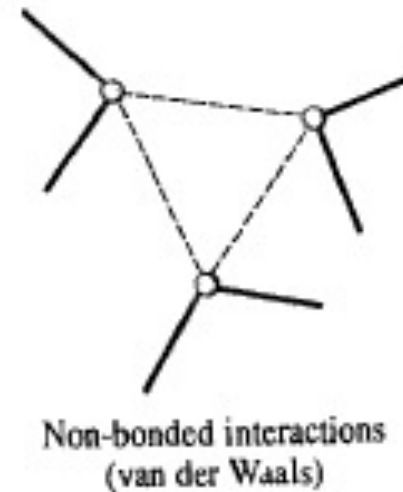
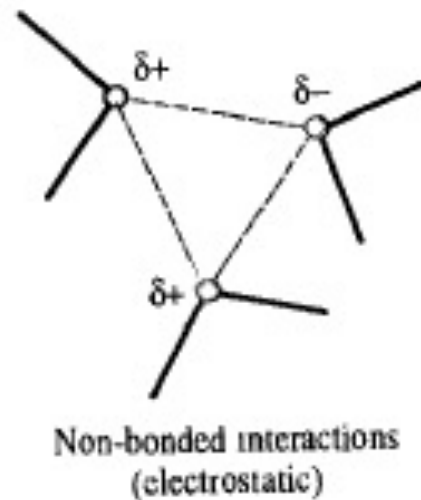
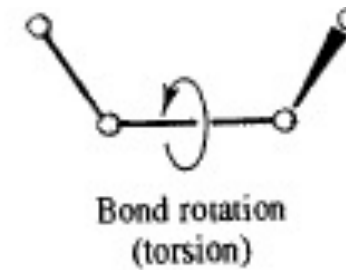
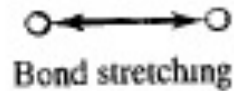
Method	Initial refinement (Av. gradient $<1 \text{ kcal } \text{Å}^{-2}$)		Stringent minimisation (Av. gradient $<0.1 \text{ kcal } \text{Å}^{-2}$)	
	CPU time (s)	Number of iterations	CPU time (s)	Number of iterations
Steepest descents	67	98	1405	1893
Conjugate gradients	149	213	257	367

Concept of force-field

$$\vec{F} = m\vec{a}$$

$$\mathbf{F}_i = -\nabla_i V_B$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$



Concept of force-field

$$\begin{aligned} \mathcal{V}(\mathbf{r}^N) = & \sum_{\text{bonds}} \frac{k_i}{2} (l_i - l_{i,0})^2 + \sum_{\text{angles}} \frac{k_i}{2} (\theta_i - \theta_{i,0})^2 + \sum_{\text{torsions}} \frac{V_n}{2} (1 + \cos(n\omega - \gamma)) \\ & + \sum_{i=1}^N \sum_{j=i+1}^N \left(4\epsilon_{ij} \left[\left(\frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}} \right)^6 \right] + \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} \right) \end{aligned}$$

Force-field examples

OPLS: 12-6 LJ potential

J. Am. Chem. Soc. **1996**, *118*, 11225–11236

11225

Development and Testing of the OPLS All-Atom Force Field on Conformational Energetics and Properties of Organic Liquids

William L. Jorgensen,* David S. Maxwell, and Julian Tirado-Rives

Contribution from the Department of Chemistry, Yale University, New Haven, Connecticut 06520-8107

Received June 27, 1996. Revised Manuscript Received September 5, 1996[Ⓞ]

Abstract: The parametrization and testing of the OPLS all-atom force field for organic molecules and peptides are described. Parameters for both torsional and nonbonded energetics have been derived, while the bond stretching and angle bending parameters have been adopted mostly from the AMBER all-atom force field. The torsional parameters were determined by fitting to rotational energy profiles obtained from ab initio molecular orbital calculations at the RHF/6-31G*/RHF/6-31G* level for more than 50 organic molecules and ions. The quality of the fits was high with average errors for conformational energies of less than 0.2 kcal/mol. The force-field results for molecular structures are also demonstrated to closely match the ab initio predictions. The nonbonded parameters were developed in conjunction with Monte Carlo statistical mechanics simulations by computing thermodynamic and structural properties for 34 pure organic liquids including alkanes, alkenes, alcohols, ethers, acetals, thiols, sulfides, disulfides, aldehydes, ketones, and amides. Average errors in comparison with experimental data are 2% for heats of vaporization and densities. The Monte Carlo simulations included sampling all internal and intermolecular degrees of freedom. It is found that such non-polar and monofunctional systems do not show significant condensed-phase effects on internal energies in going from the gas phase to the pure liquids.

COMPASS: 9-6 LJ potential

7338

J. Phys. Chem. B **1998**, *102*, 7338–7364

COMPASS: An ab Initio Force-Field Optimized for Condensed-Phase Applications—Overview with Details on Alkane and Benzene Compounds

H. Sun

Molecular Simulations Inc., 9685 Scranton Road, San Diego, California 92121-3752

Received: January 29, 1998; In Final Form: May 22, 1998

A general all-atom force field for atomistic simulation of common organic molecules, inorganic small molecules, and polymers was developed using state-of-the-art ab initio and empirical parametrization techniques. The valence parameters and atomic partial charges were derived by fitting to ab initio data, and the van der Waals (vdW) parameters were derived by conducting MD simulations of molecular liquids and fitting the simulated cohesive energies and equilibrium densities to experimental data. The combined parametrization procedure significantly improves the quality of a general force field. Validation studies based on large number of isolated molecules, molecular liquids and molecular crystals, representing 28 molecular classes, show that the present force field enables accurate and simultaneous prediction of structural, conformational, vibrational, and thermophysical properties for a broad range of molecules in isolation and in condensed phases. Detailed results of the parametrization and validation for alkane and benzene compounds are presented.

Force-field examples

DREIDING: 12-10 LJ potential

J. Phys. Chem. **1990**, *94*, 8897–8909

8897

DREIDING: A Generic Force Field for Molecular Simulations

Stephen L. Mayo, Barry D. Olafson, and William A. Goddard III*,¹

BioDesign, Inc., 199 South Los Robles (Suite 540), Pasadena, California 91101 (Received: October 2, 1989; In Final Form: February 2, 1990)

We report the parameters for a new generic force field, DREIDING, that we find useful for predicting structures and dynamics of organic, biological, and main-group inorganic molecules. The philosophy in DREIDING is to use general force constants and geometry parameters based on simple hybridization considerations rather than individual force constants and geometric parameters that depend on the particular combination of atoms involved in the bond, angle, or torsion terms. Thus all bond distances are derived from atomic radii, and there is only one force constant each for bonds, angles, and inversions and only six different values for torsional barriers. Parameters are defined for all possible combinations of atoms and new atoms can be added to the force field rather simply. This paper reports the parameters for the “nonmetallic” main-group elements (B, C, N, O, F columns for the C, Si, Ge, and Sn rows) plus H and a few metals (Na, Ca, Zn, Fe). The accuracy of the DREIDING force field is tested by comparing with (i) 76 accurately determined crystal structures of organic compounds involving H, C, N, O, F, P, S, Cl, and Br, (ii) rotational barriers of a number of molecules, and (iii) relative conformational energies and barriers of a number of molecules. We find excellent results for these systems.

CFF: 9-6 LJ potential

J. Am. Chem. Soc. **1994**, *116*, 2978–2987

An ab Initio CFF93 All-Atom Force Field for Polycarbonates

Huai Sun, Stephen J. Mumby, Jon R. Maple, and Arnold T. Hagler*

Contribution from the Biosym Technologies, Inc., 9685 Scranton Road, San Diego, California 92121

*Received March 29, 1993. Revised Manuscript Received November 29, 1993**

Abstract: An all-atom CFF93 force field for polycarbonates based on *ab initio* calculations is reported. Force field parameters are derived by fitting to quantum mechanical total energies, first and second derivatives of total energies, and electrostatic potentials, all generated from *ab initio* quantum mechanical calculations on model compounds at HF/6-31G* level of theory. Valence parameters and *ab initio* charges are then scaled to correct for differences between experiment and the Hartree–Fock approximation. The van der Waals parameters and the scaling factors for atomic partial charges are determined from crystal structures. Based on the force field, molecular mechanics calculations are performed for several model compounds, and the results are compared with experimental values and with the results of the *ab initio* calculations.

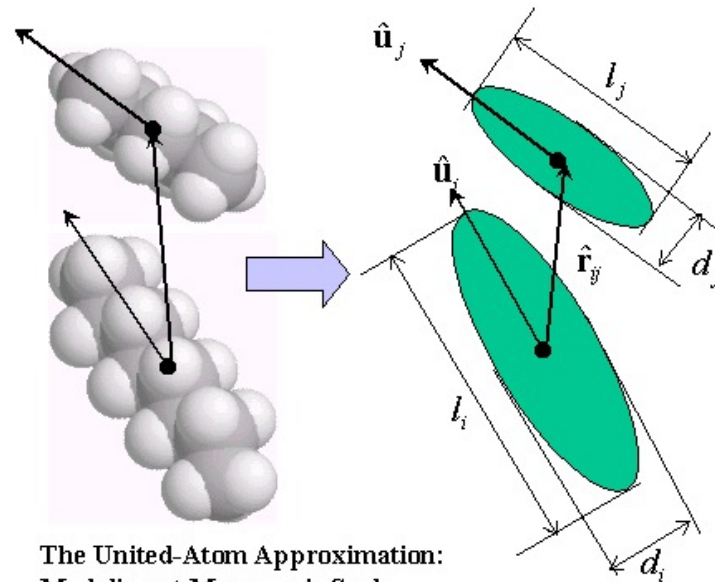
Coarse-graining and coarse-grained simulations

Smoothing out of the atomistic detail

Loss of degree of freedom

1) United atom representation

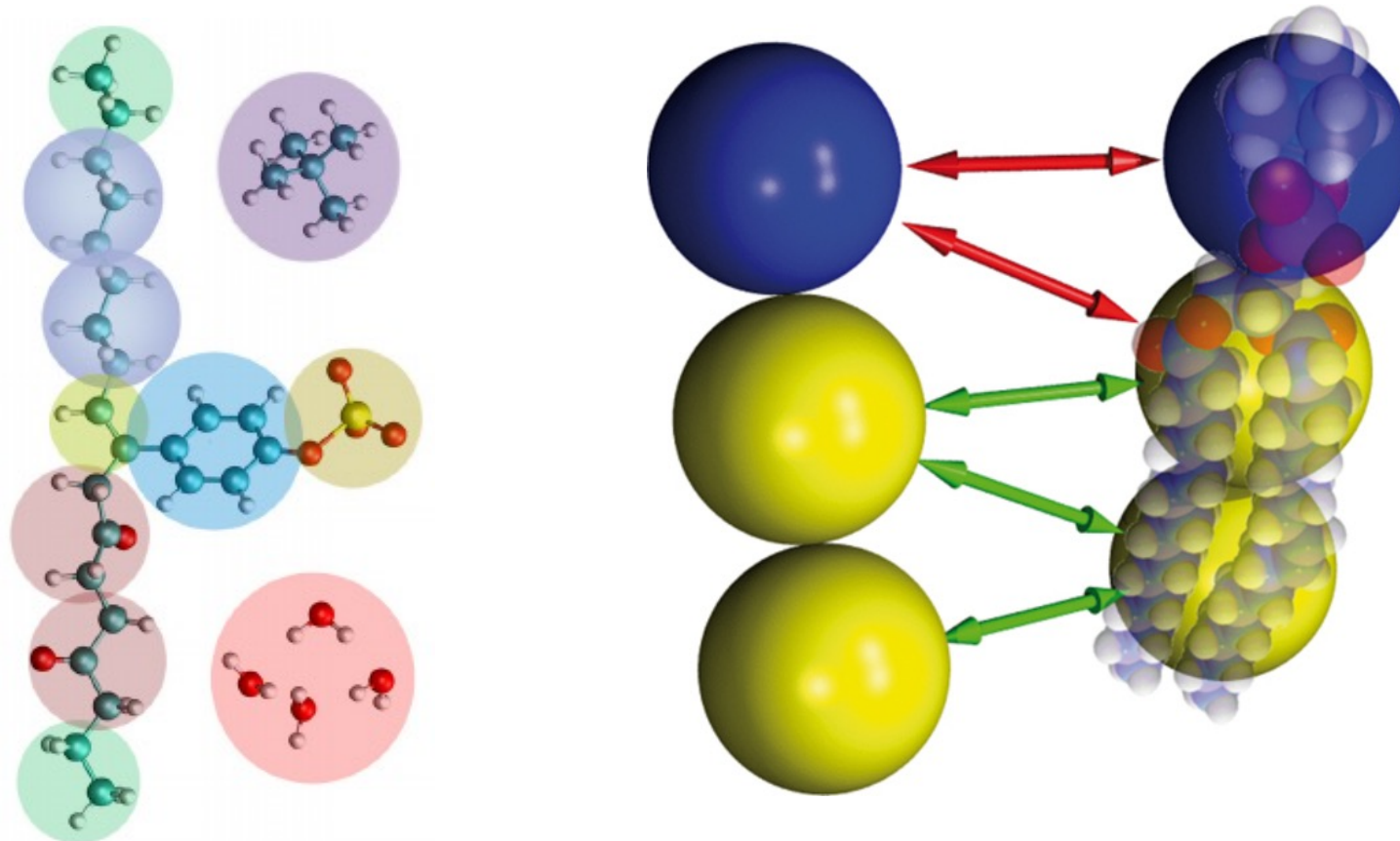
Combine small atoms such as H's with heavy atoms



Coarse-graining and coarse-grained simulations

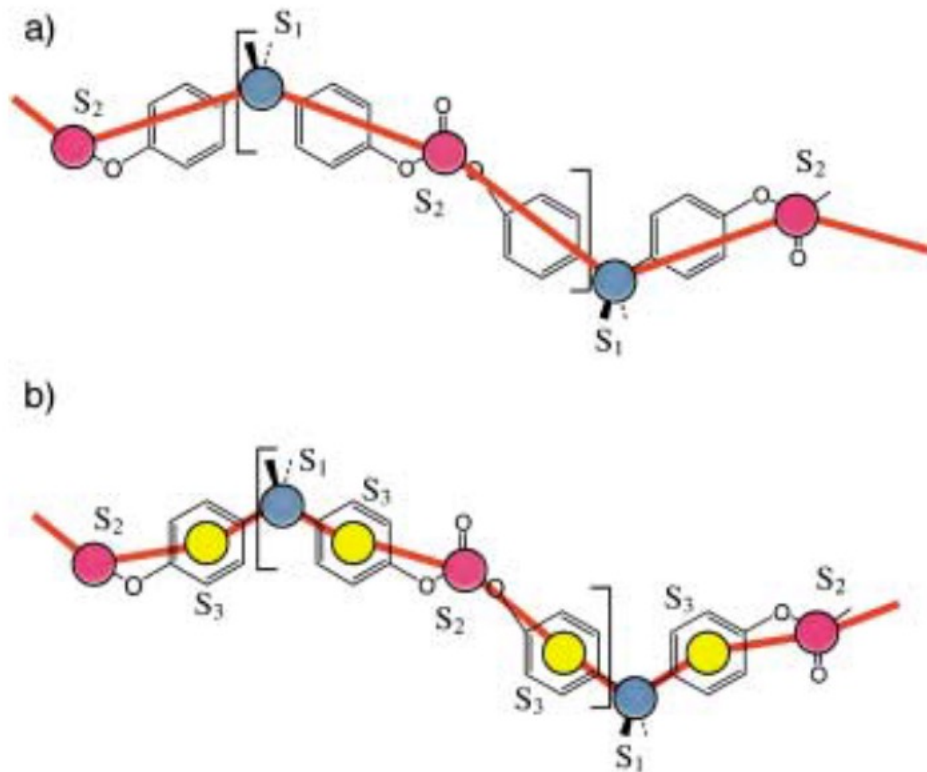
Coarse-graining based on grouping of atoms

2) Grouping atoms together (order of 10:1)



Coarse-graining and coarse-grained simulations

Different levels of coarse-graining



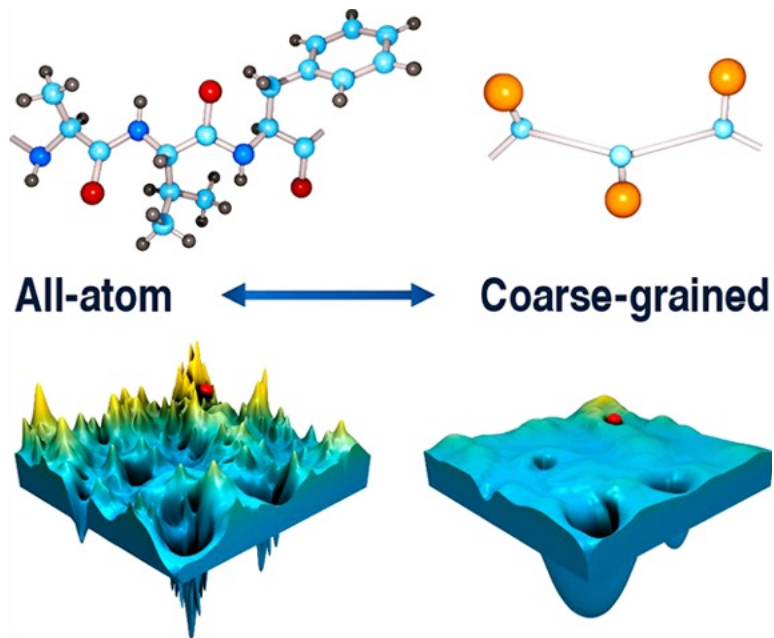
Coarse-graining of bisphenol-A polycarbonate

a) 2:1 scheme (two super-atoms per chemical repeat unit)

b) 4:1 scheme

Coarse-grained potential

Integrated out atomistic detail yields smoothed potential energy surface



Advantages

- Less rugged potential energy surface
- Simulating longer time and length scales
- Faster equilibration of polymeric (or complex soft matter) systems

Disadvantages

- Poor performance in sampling local minima
- Less ability to compute energetic properties
- Loss of ability to perform nonequilibrium dynamics
- Fail to predict some (e.g. mechanical) properties

Coarse-grained potential

Smoothed out atomistic detail leads to 'effective pair-wise' interactions

Interaction parameters are not differentiated based on different systems for atomistic simulations

A generic all-atom (AA) model describes features common to all polymers

However,

A coarse-grained model is different from a generic AA model

Example: A CG model of polystyrene is different than polyisobutylene melt

Coarse-grained potential

How does the ‘effective pair-wise potential’ differ?

Accelerated equilibration of polymer melts by time-coarse-graining

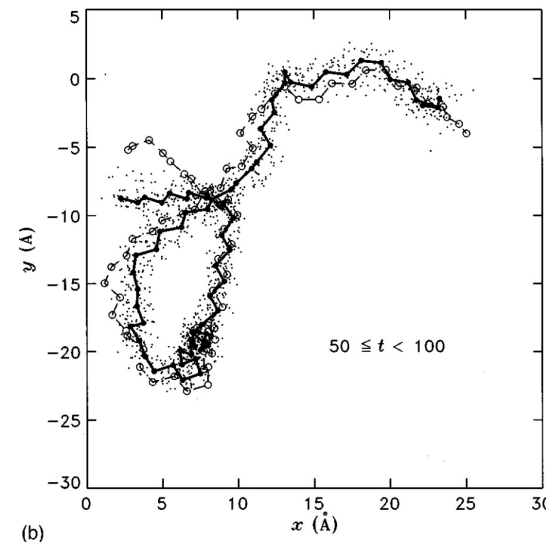
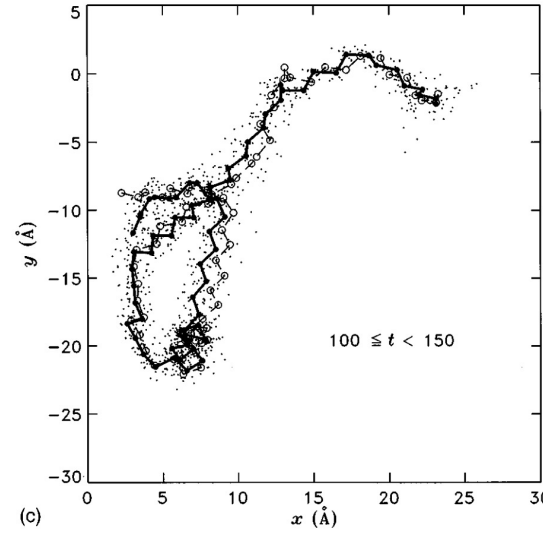
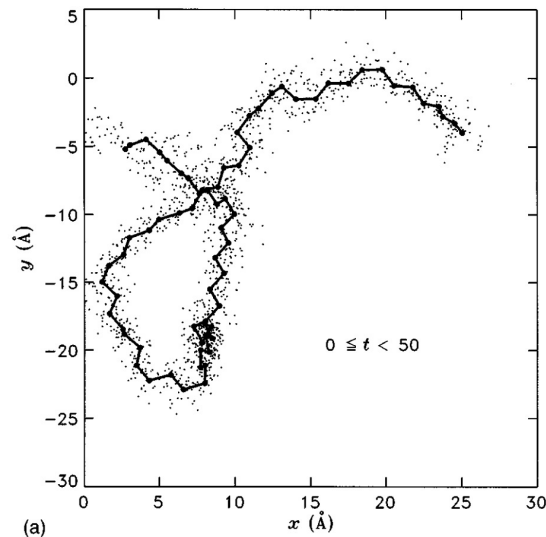
Bruce M. Forrest and Ulrich W. Suter

Institut für Polymere, Eidgenössische Technische Hochschule (ETH), CH-8092, Zürich, Switzerland

(Received 8 November 1994; accepted 31 January 1995)

An effective-potential approach is presented for improving sampling efficiency in simulations of atomistically detailed models of dense long-chain liquids. The motion of atoms on short-time scales is described as rapid fluctuations about the slowly moving mean conformations of the chain molecules. The distribution of these fluctuations is approximated by that of isotropic elastic motion. The interactions between nonbonded pairs of atoms are preaveraged over this distribution and a much softer, effective interaction is obtained, allowing a correspondingly faster exploration of configuration space. The approximate sampling scheme is then tested on two model systems of united-atom liquid hydrocarbons—a melt of twenty C_{24} chains and one of ten C_{71} chains. In the C_{24} melt, where a comparison with fully equilibrated samples from rigorous algorithms is possible, the preaveraging is shown to produce an improvement in sampling efficiency of up to an order of magnitude at the expense of only a moderate loss in accuracy. The method is then applied to the C_{71} melt, where currently available, rigorous sampling algorithms fail to produce thorough equilibration. Also in this case, the preaveraging method proves to significantly enhance the sampling speed while producing satisfactory distributions of observables. © 1995 American Institute of Physics.

Coarse-grained potential



Time averaging of the fast dynamics is performed along with the slow motion of the whole chain

$$r_i(t_0 + t) = R_i(t_0) + \Delta_i(t), \quad 0 \leq t < t_{av}$$

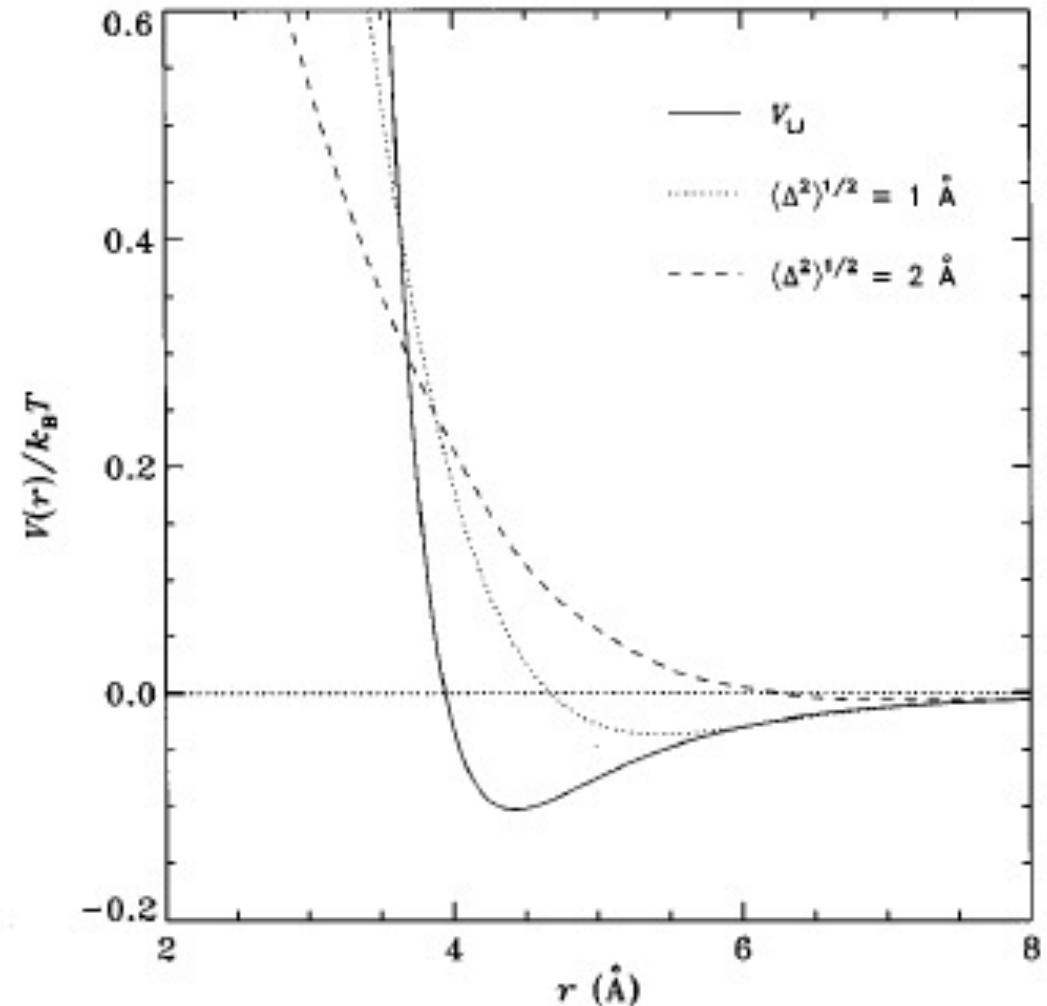
$\Delta_i(t)$ = fluctuation of atom i

R_i = mean position, where atoms fluctuate around and constant for duration of t_{av}

Coarse-grained potential

Level of average fluctuations is important!

Integration out of the fast vibrations
leads to a softer effective pair-wise
potential



Dissipative Particle Dynamics DPD

First developed by Hoogerbrugge and Koelman to simulate the 'hydrodynamic phenomena'

Particle-based method replacing Lattice Gas automata

Proper description of complex fluids with computational efficiency

Presented as a simple model contains dissipation that conserves momentum

EUROPHYSICS LETTERS

Europhys. Lett., **19** (3), pp. 155-160 (1992)

1 June 1992

Simulating Microscopic Hydrodynamic Phenomena with Dissipative Particle Dynamics.

P. J. HOOGERBRUGGE and J. M. V. A. KOELMAN

Shell Research B.V. - P.O.Box 60, 2280 AB Rijswijk, The Netherlands

(received 19 February 1992; accepted in final form 29 April 1992)

PACS. 02.70 - Computational techniques.

PACS. 51.10 - Kinetic and transport theory.

Abstract. - We present a novel method for simulating hydrodynamic phenomena. This particle-based method combines features from molecular dynamics and lattice-gas automata. It is shown theoretically as well as in simulations that a quantitative description of isothermal Navier-Stokes flow is obtained with relatively few particles. Computationally, the method is much faster than molecular dynamics, and at the same time it is much more flexible than lattice-gas automata schemes.

DPD

Español and Warren later reformulized the total force term as

$$\dot{\mathbf{p}}_i = \sum_{j \neq i} \mathbf{F}_{ij}^C + \sum_{j \neq i} \mathbf{F}_{ij}^D + \sum_{j \neq i} \mathbf{F}_{ij}^R,$$

where,

$$\mathbf{F}_{ij}^D = -\gamma\omega_D(r_{ij})(\mathbf{e}_{ij} \cdot \mathbf{v}_{ij})\mathbf{e}_{ij}$$
$$\mathbf{F}_{ij}^R = \sigma\omega_R(r_{ij})\mathbf{e}_{ij}\zeta_{ij},$$

Stochastic white noise term

satisfying $\zeta_{ij} = -\zeta_{ji}$ to conserve momentum

Holds fluctuation-dissipation theorem such that (similar to Brownian motion)

$$\sigma = (2k_B T \gamma)^{1/2}$$

Statistical Mechanics of Dissipative Particle Dynamics.

P. ESPAÑOL (*)^(§) and P. WARREN (**)

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(received 22 December 1994; accepted in final form 30 March 1995)

PACS. 02.70-c - Computational techniques.

PACS. 51.10+y - Kinetic and transport theory of gases.

Abstract. - The stochastic differential equations corresponding to the updating algorithm of Dissipative Particle Dynamics (DPD), and the corresponding Fokker-Planck equation are derived. It is shown that a slight modification to the algorithm is required before the Gibbs distribution is recovered as the stationary solution to the Fokker-Planck equation. The temperature of the system is then directly related to the noise amplitude by means of a fluctuation-dissipation theorem. However, the correspondingly modified, discrete DPD algorithm is only found to obey these predictions if the length of the time step is sufficiently reduced. This indicates the importance of time discretisation in DPD.

DPD (Dissipative Particle Dynamics)

In summary

Conservative, **R**andom and **D**issipative forces

C force characterizes the equilibrium structure

R and **D** forces are coupled and act as a thermostat

$$\begin{aligned}\mathbf{F}^T &= m\vec{a} \\ &= \mathbf{F}^C + \mathbf{F}^R + \mathbf{F}^D\end{aligned}$$

$$\mathbf{r}_i(t + \Delta t) = \mathbf{r}_i(t) + \Delta t \mathbf{v}_i(t) + \frac{1}{2}(\Delta t)^2 \mathbf{f}_i(t),$$

$$\tilde{\mathbf{v}}_i(t + \Delta t) = \mathbf{v}_i(t) + \lambda \Delta t \mathbf{f}_i(t),$$

$$\mathbf{f}_i(t + \Delta t) = \mathbf{f}_i(\mathbf{r}(t + \Delta t), \tilde{\mathbf{v}}(t + \Delta t)),$$

$$\mathbf{v}_i(t + \Delta t) = \mathbf{v}_i(t) + \frac{1}{2}\Delta t(\mathbf{f}_i(t) + \mathbf{f}_i(t + \Delta t)).$$

For $\lambda = \frac{1}{2}$ Verlet algorithm is recovered.

How to compute the parameters?

Dissipative particle dynamics: Bridging the gap between atomistic and mesoscopic simulation

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United Kingdom*

(Received 27 March 1997; accepted 16 June 1997)

We critically review dissipative particle dynamics (DPD) as a mesoscopic simulation method. We have established useful parameter ranges for simulations, and have made a link between these parameters and χ -parameters in Flory-Huggins-type models. This is possible because the equation of state of the DPD fluid is essentially quadratic in density. This link opens the way to do large scale simulations, effectively describing millions of atoms, by firstly performing simulations of molecular fragments retaining all atomistic details to derive χ -parameters, then secondly using these results as input to a DPD simulation to study the formation of micelles, networks, mesophases and so forth. As an example application, we have calculated the interfacial tension σ between homopolymer melts as a function of χ and N and have found a universal scaling collapse when $\sigma/\rho k_B T \chi^{0.4}$ is plotted against χN for $N > 1$. We also discuss the use of DPD to simulate the dynamics of mesoscopic systems, and indicate a possible problem with the timescale separation between particle diffusion and momentum diffusion (viscosity). © 1997 American Institute of Physics.

[S0021-9606(97)51335-3]

Groot-Warren approach

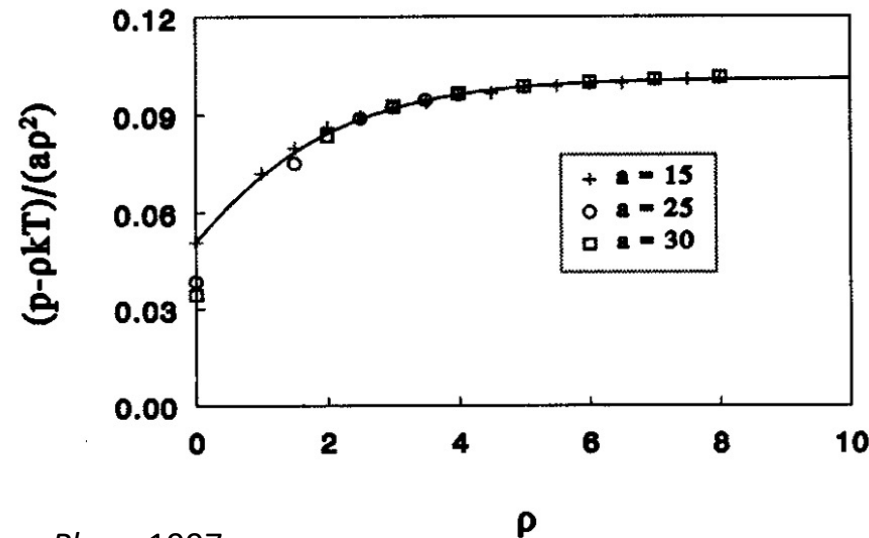
Conservative force is computed from non-bonded DPD interaction parameter a_{ij}

$$\mathbf{F}_{ij}^C = \begin{cases} a_{ij}(1-r_{ij})\hat{\mathbf{r}}_{ij} & (r_{ij} < 1) \\ 0 & (r_{ij} \geq 1) \end{cases}$$

How to choose the 'repulsion parameter'?

Building a DPD EoS!

Vary the density and plot the excess pressure



Groot and Warren, *J. Chem. Phys.*, 1997.

For a sufficiently large density
for $\rho > 2$

$$p = \rho k_B T + \alpha a \rho^2 (\alpha = 0.101 \pm 0.001)$$

Non-bonded DPD interaction strength

Groot-Warren approach

Mapping onto Flory-Huggins theory of polymers

Continuous version of the lattice model

Internal energy is described as the perturbation from ideal mixing

Free energy per lattice site

$$\frac{F}{k_B T} = \frac{\phi_A}{N_A} \ln \phi_A + \frac{\phi_B}{N_B} \ln \phi_B + \chi \phi_A \phi_B$$

From $p = \rho k_B T + \alpha a \rho^2$ ($\alpha = 0.101 \pm 0.001$) for a two-component mixture

$$\begin{aligned} \frac{f_V}{k_B T} = & \frac{\rho_A}{N_A} \ln \rho_A + \frac{\rho_B}{N_B} \ln \rho_B - \frac{\rho_A}{N_A} - \frac{\rho_B}{N_B} \\ & + \frac{\alpha(a_{AA}\rho_A^2 + 2a_{AB}\rho_A\rho_B + a_{BB}\rho_B^2)}{k_B T} \end{aligned}$$

Mapping onto Flory-Huggins theory

A χ -parameter mapping $f_V/(\rho_A + \rho_B) = F$ (Flory-Huggins) yields

$$\chi = \frac{2\alpha(a_{AB} - a_{AA})(\rho_A + \rho_B)}{k_B T}$$

We can have a reasonable estimation for the a_{AA} from $\kappa^{-1} = \frac{1}{nk_B T \kappa_T} = \frac{1}{k_B T} \left(\frac{\partial p}{\partial n} \right)_T$

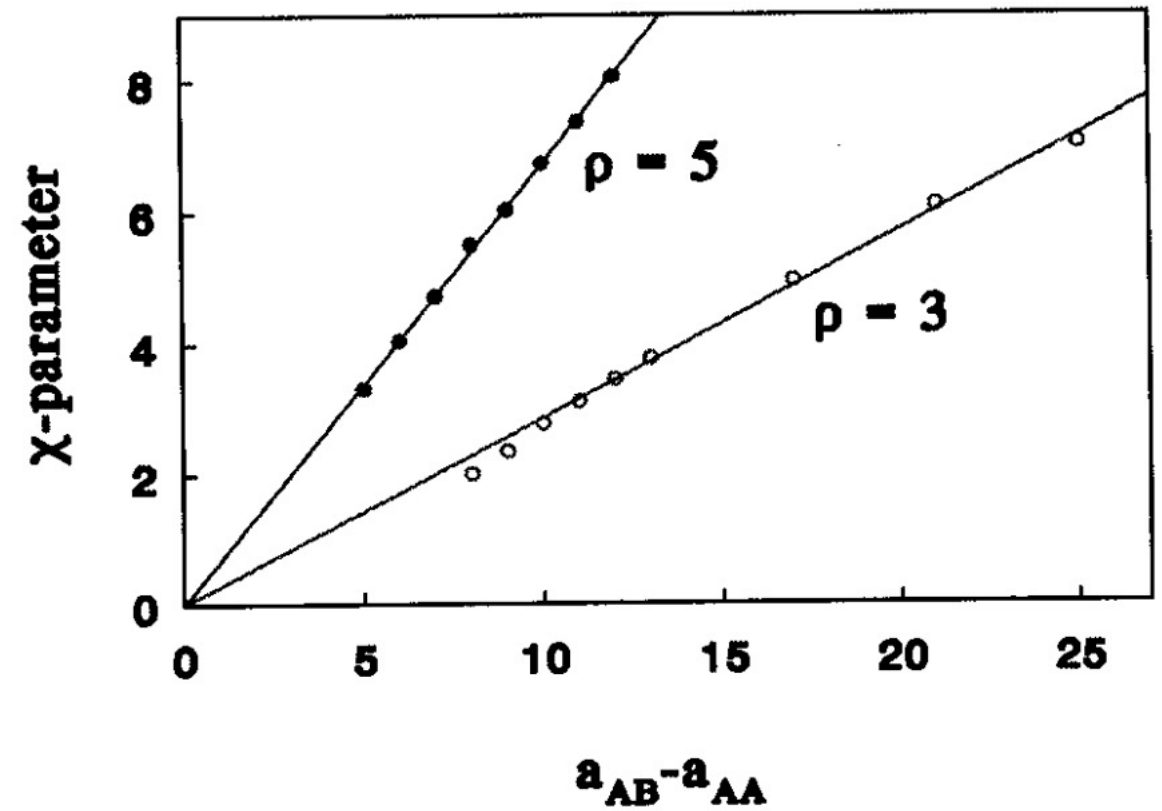
$$\kappa^{-1} = 15.9835 \quad \text{Experimental data}$$

$$\kappa^{-1} = 1 + 2\alpha a \rho / k_B T \approx 1 + 0.2 a \rho / k T$$

$$\kappa_{\text{water}}^{-1} \approx 16$$

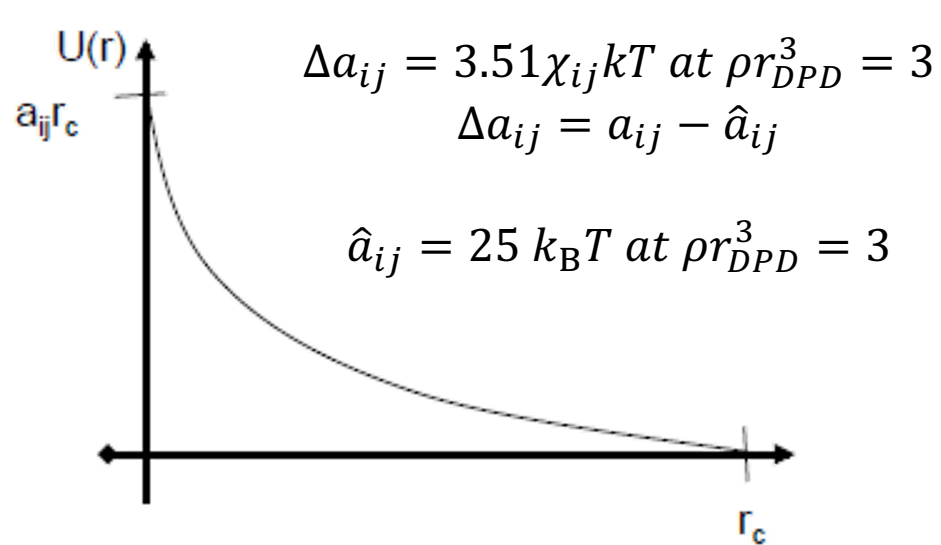
$$a \rho / k_B T \approx 75 \quad a = 25 k_B T$$

A linear relationship between a 's and χ -parameter is identified



DPD (Dissipative Particle Dynamics)

Non-bonded **soft & purely repulsive** pair-potential

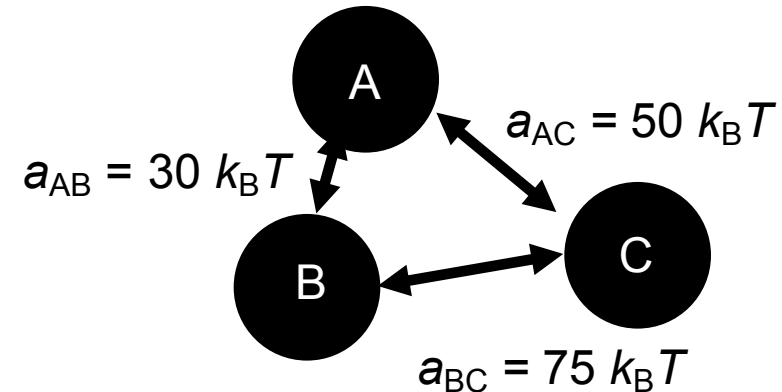


Relative attraction of beads based on preferential repulsions

$$a_{ij}(T) = \begin{cases} a_{ii} + 3.497 k_B T \chi_{ij}(T) (\rho = 3) \\ a_{ii} + 1.451 k_B T \chi_{ij}(T) (\rho = 5) \end{cases}$$

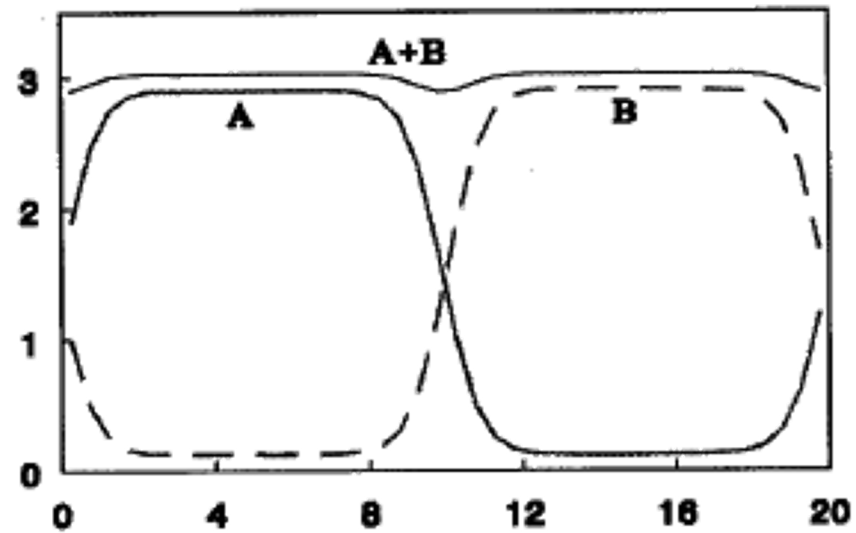
$$\chi = \frac{V_m}{RT} [(\delta_1 - \delta_2)^2]$$

$$a_{AA} = a_{BB} = a_{CC} = 25 k_B T$$



DPD – Equal bead sizes

Assumption: Equal-size beads

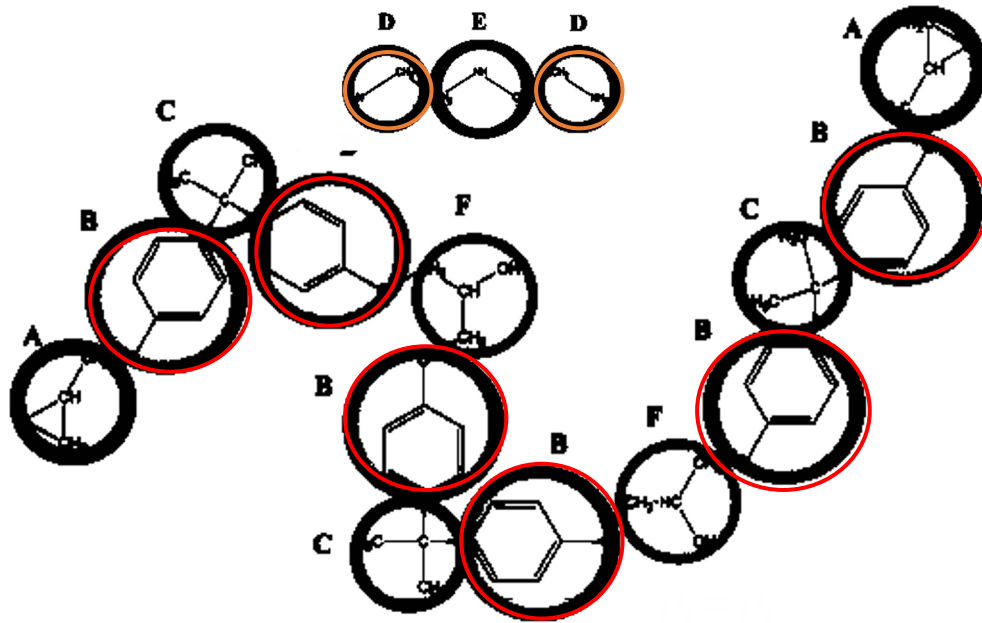


$$\rho_i = \rho_j$$

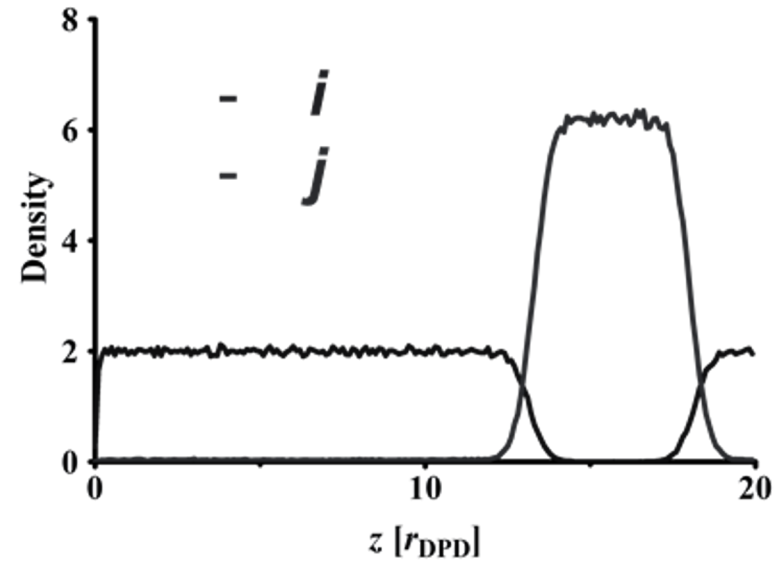
$$\rho_{i, \text{pure}} = \rho_{j, \text{pure}}$$

$$a_{ii} = a_{jj}$$

DPD – Equal bead sizes



$V_{\text{bead},B} (\text{\AA}^3)$	$V_{\text{bead},D} (\text{\AA}^3)$
151	50



$$p_i = p_j$$

$$\rho_{i, \text{pure}} \neq \rho_{j, \text{pure}}$$

$$a_{ii} \neq a_{jj}$$

DPD – Different bead sizes

DPD EoS

$$p = \rho kT + \alpha \rho^2 r_{DPD}^3, \alpha = 0.101 \quad \boxed{a_{ii} = \frac{p - \rho_{i,pure} kT}{\alpha \rho_{i,pure}^2 r_{DPD}^3}}$$

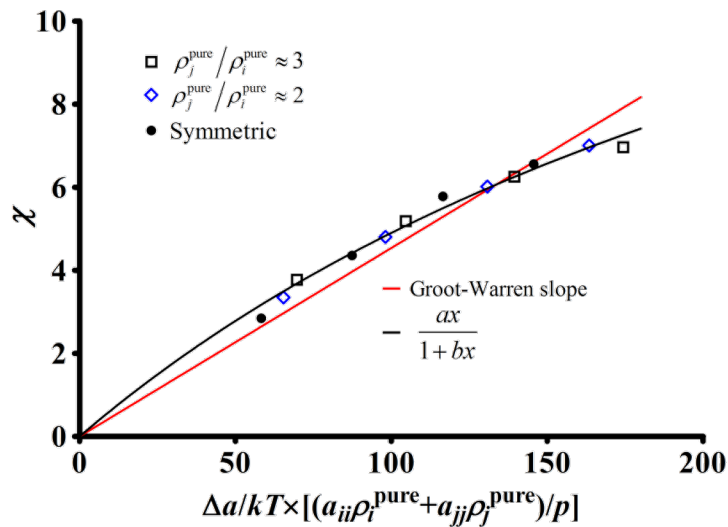
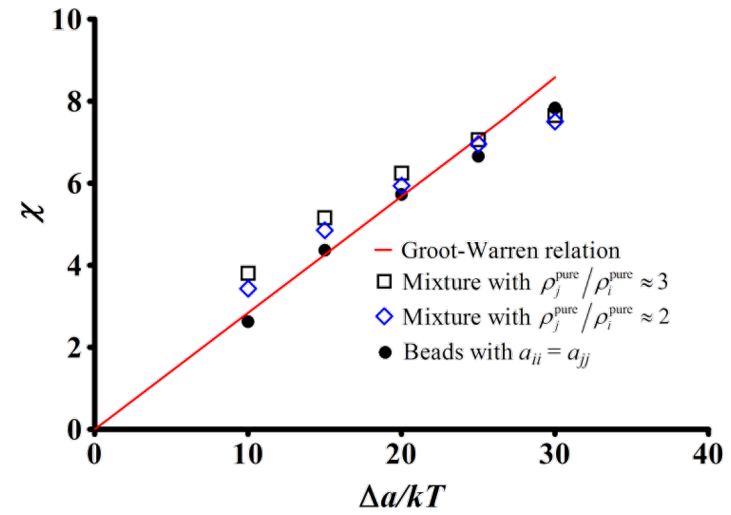
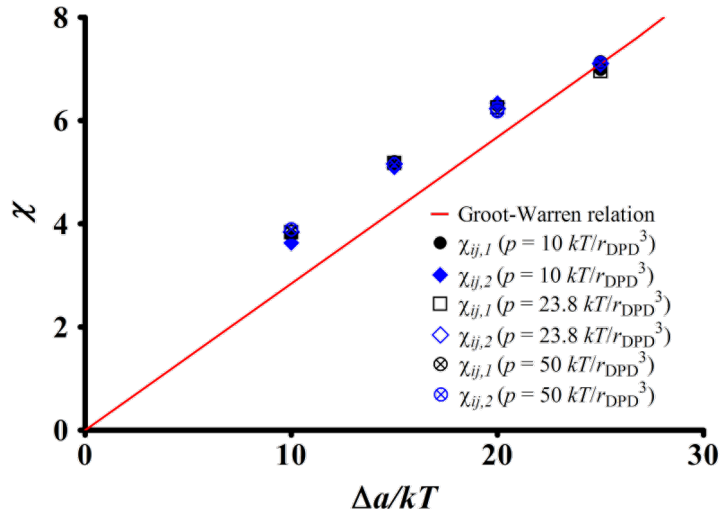
$$\rho_{i,pure}^2 a_{ii} \approx \rho_{j,pure}^2 a_{jj} \propto p \quad \rho^{-1} = \bar{V} = \sum x_i \rho_{i,pure}^{-1} \approx \sqrt{\frac{\alpha r_{DPD}^3}{p}} \sum x_i \sqrt{a_{ii}}$$

$$p = \rho kT + \alpha \rho^2 r_{DPD}^3 A(x) \quad A(x) = \sum x_i x_j \hat{a}_{ij}$$

$$\begin{aligned} A(x) &= \sum x_i x_j \hat{a}_{ij} = \left(\sum x_i \sqrt{a_{ii}} \right)^2 + \sum x_i x_j \left(\hat{a}_{ij} - \sqrt{a_{ii} a_{jj}} \right) \\ &= \frac{p}{\alpha r_{DPD}^3 \rho^2} + \sum x_i x_j \left(\hat{a}_{ij} - \sqrt{a_{ii} a_{jj}} \right) \end{aligned}$$

$$\boxed{\hat{a}_{ij} = \sqrt{a_{ii} a_{jj}}}$$

Mapping on Flory Huggins parameter



$$\phi_i = \frac{x_i \rho_{i,\text{pure}}^{-1}}{\sum_j x_j \rho_{j,\text{pure}}^{-1}}$$

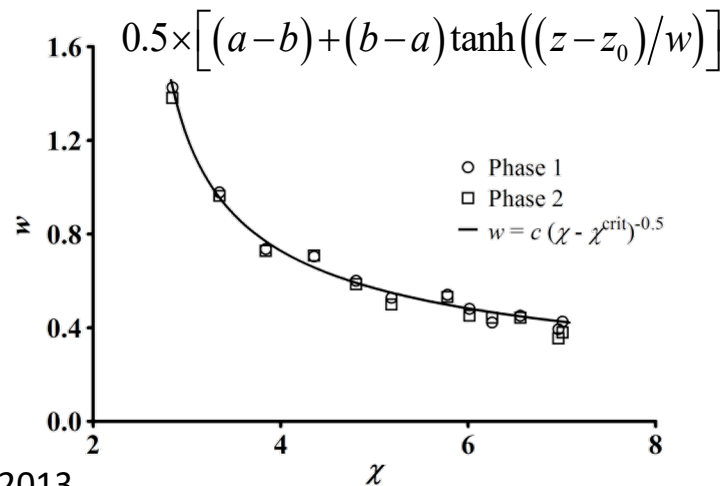
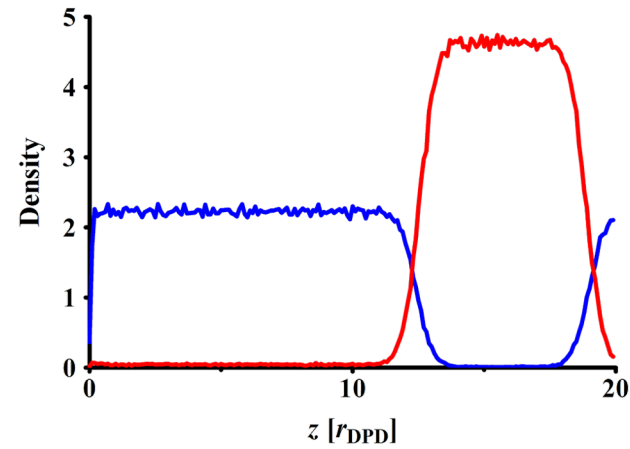
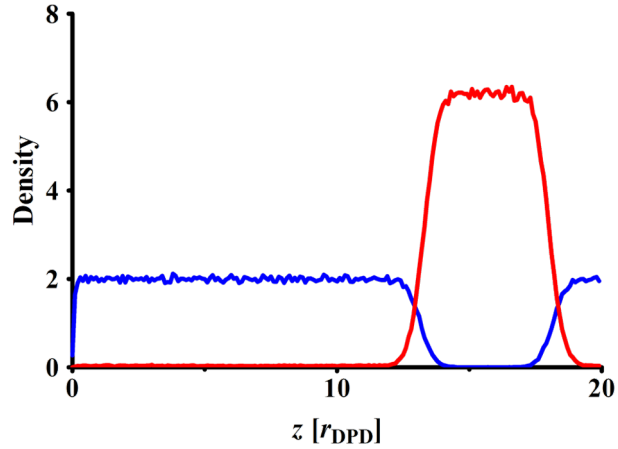
$$\Delta G_{\text{mix}}/kT = \sum_i \phi_i \ln \phi_i + \sum_i \sum_{j>i} \chi_{ij} \phi_i \phi_j$$

$$\chi_{ij,1} = \frac{\ln[\phi_{j1}/\phi_{i1}]}{1-2\phi_{i1}}, \quad \chi_{ij,2} = \frac{\ln[\phi_{j2}/\phi_{i2}]}{1-2\phi_{i2}}$$

$$a_{ij} = \hat{a}_{ij} + \frac{p}{0.0454(a_{ii}\rho_i^{\text{pure}} + a_{jj}\rho_j^{\text{pure}})} \chi_{ij} kT$$

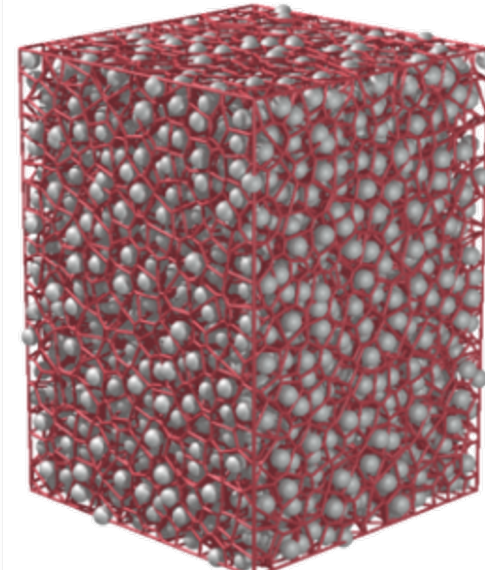
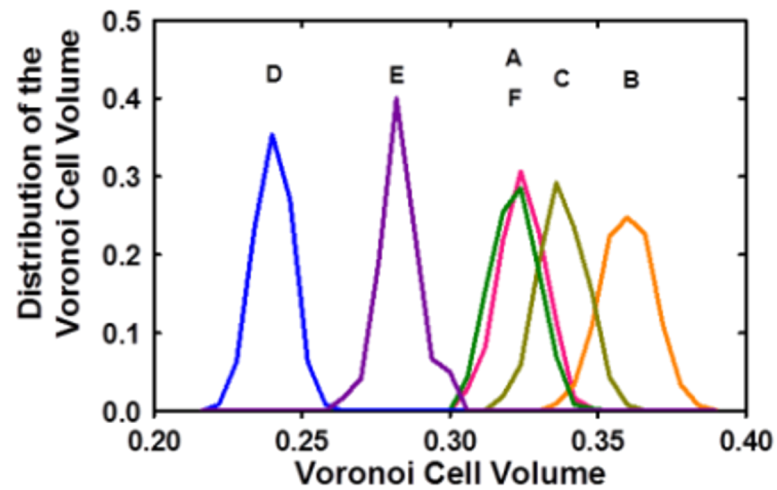
Validation

Interface width $\sim \chi$ -parameter

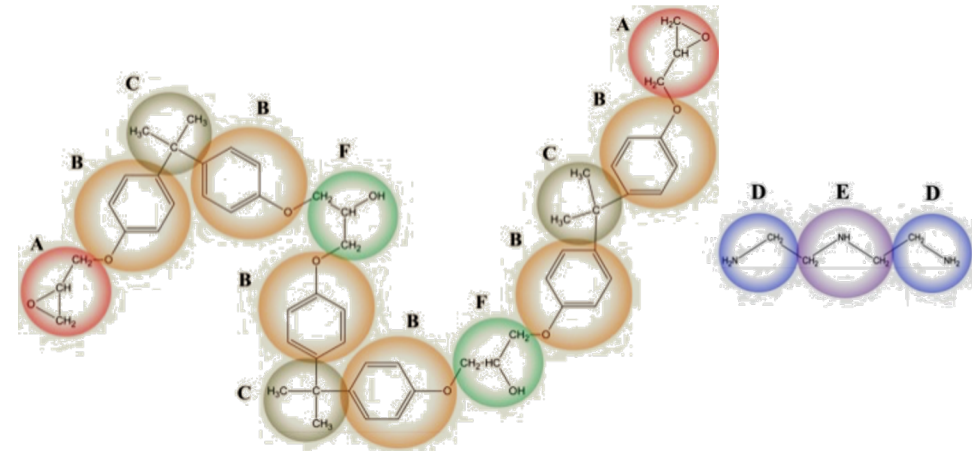


$$\chi^{\text{crit}} = 2.44 \quad (2.10 \pm 0.04 \text{ in Groot-Warren})$$

DPD – Different bead sizes



Bead type	A	B	C	F	D	E
Pure liquid bead volume (r_{DPD}^3)	0.281	0.413	0.316	0.276	0.136	0.205
Volume of free beads (r_{DPD}^3)	0.313 (0.007)	0.364 (0.007)	0.327 (0.007)	0.311 (0.007)	0.257 (0.006)	0.282 (0.006)



Kacar, G. et al., *EPL*, 2013.

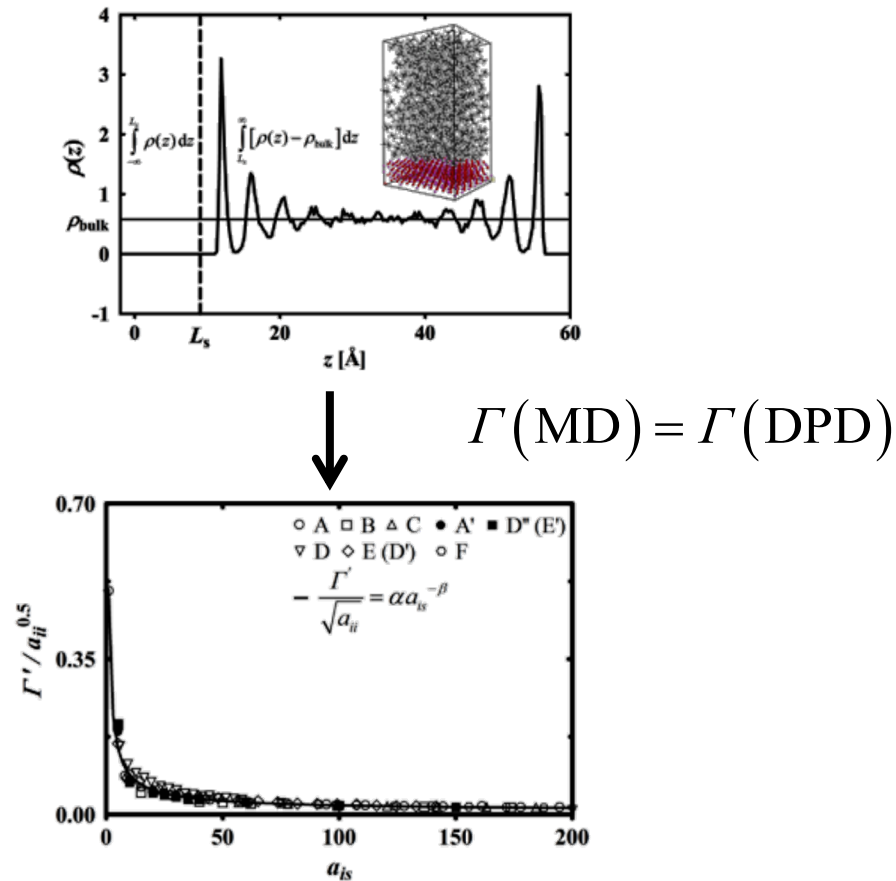
Kacar, G. et al., *Soft Matter* 2013.

Mesoscopic *interface* structure from DPD

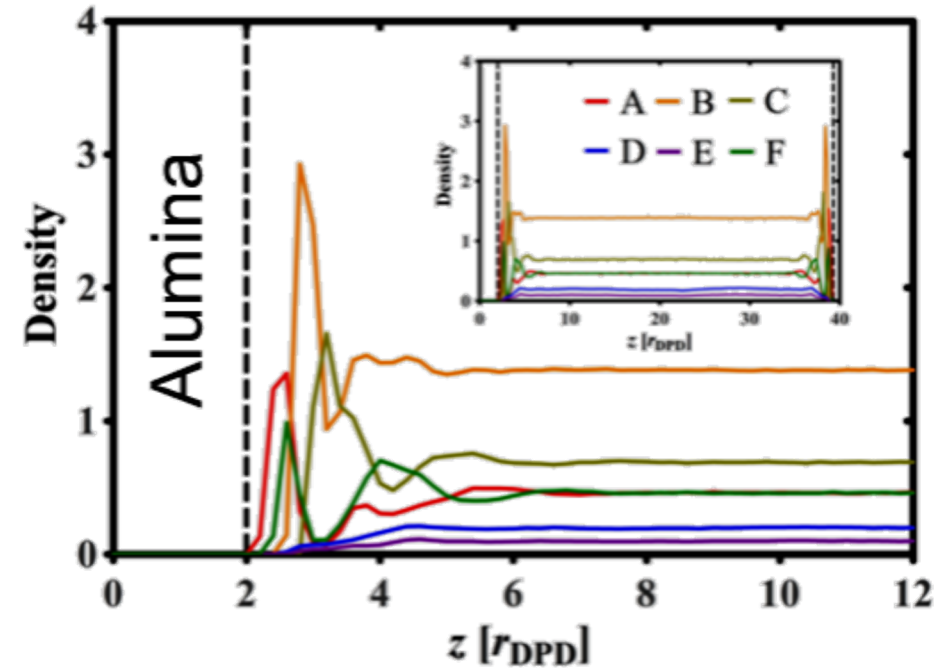
Modeling polymer-surface interactions with DPD

Coupling of atomistic and DPD scales

Quantify amount of beads adsorbed with surface excess Γ

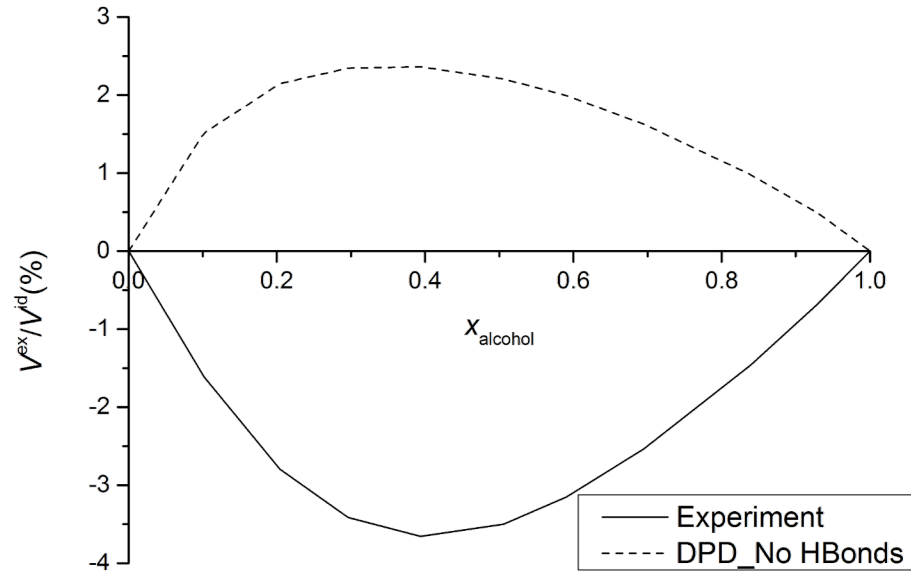


Different bead sizes are important to map MD to DPD



Hydrogen bond interactions in DPD

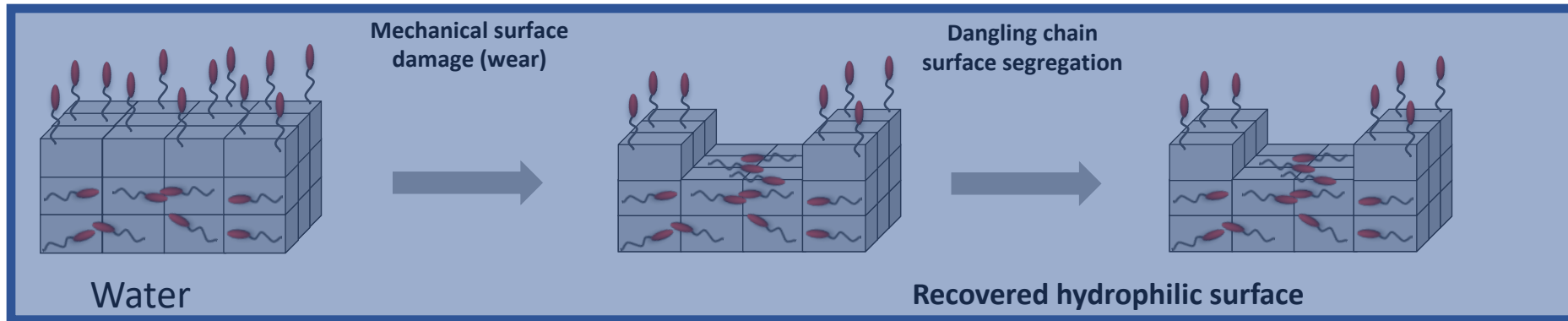
Some low M_W alcohol-water mixtures
Negative excess volume



$$V^E = \frac{x_A M_A + x_B M_B}{\rho_L} - \underbrace{x_A V_{m,A} - x_B V_{m,B}}_{\text{Ideal contribution}}$$

Modeling hydrophilic polymers with DPD

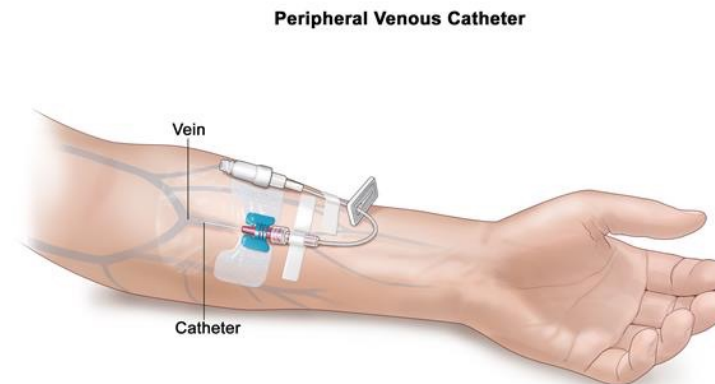
Self-replenishing hydrophilic surfaces



Dikić *et al.* *Adv. Mater.*, 2012.

Functional polymeric surfaces

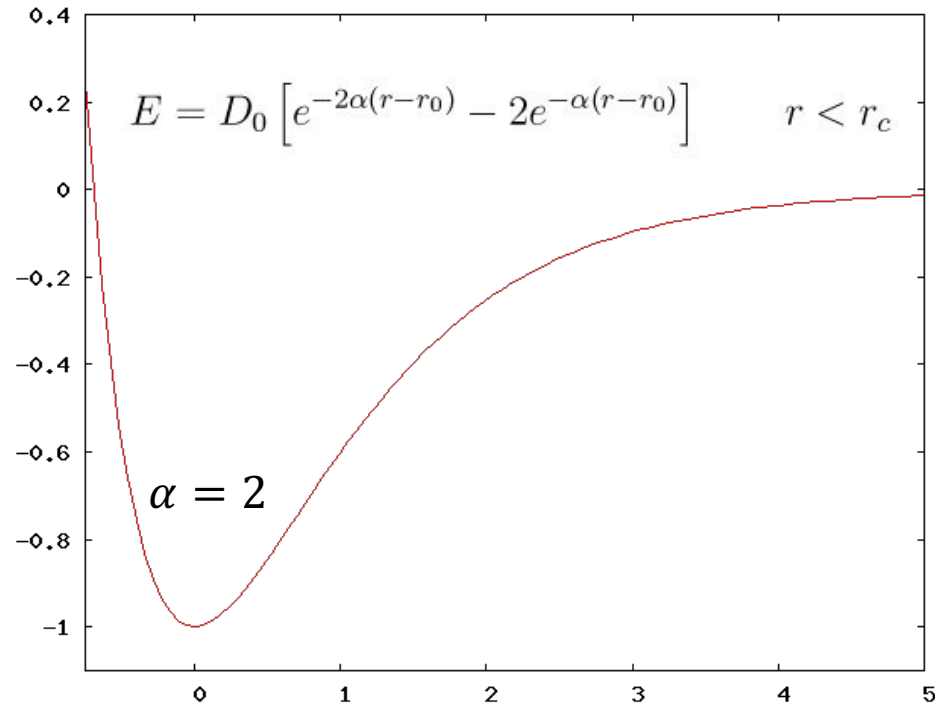
Water mimics physiological conditions



Hydrogen bonding in DPD

Hydrogen bond interactions
with Morse Potential

$$\vec{f}^T = \vec{f}^C + \vec{f}^R + \vec{f}^D + \vec{f}^{\text{Morse}}$$



Two parameters required

H-bond distance r_0

Depth of the energy well D_0

To test

Methanol– Water

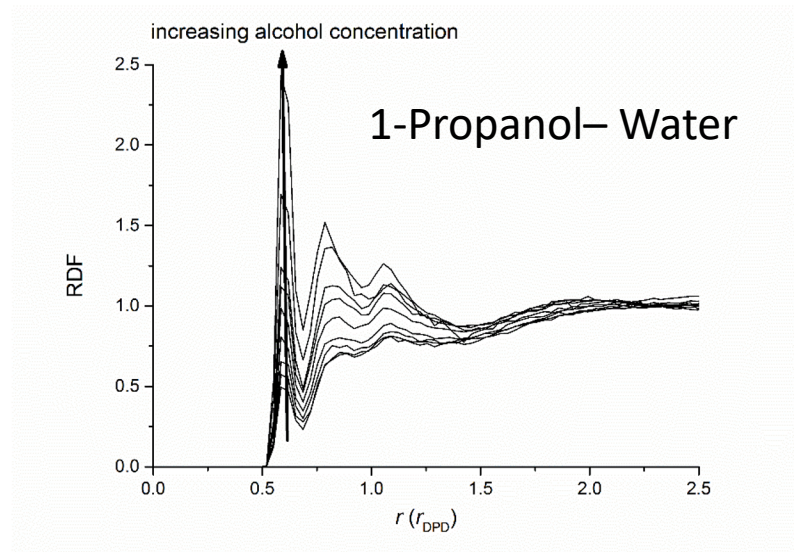
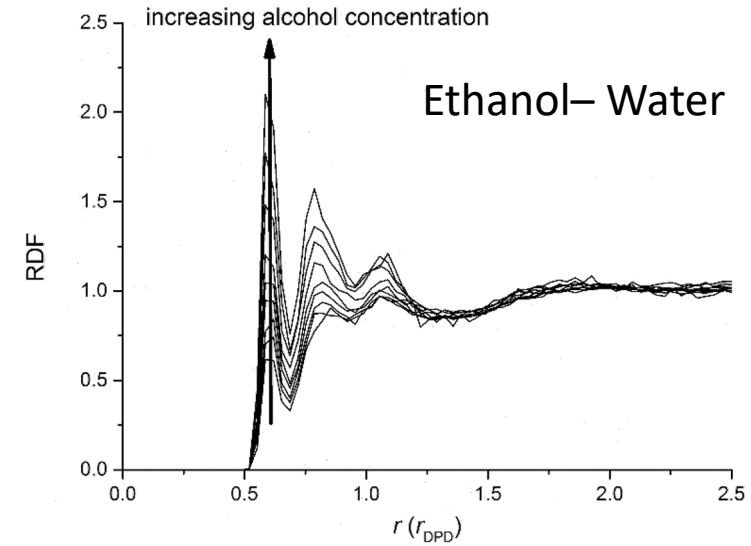
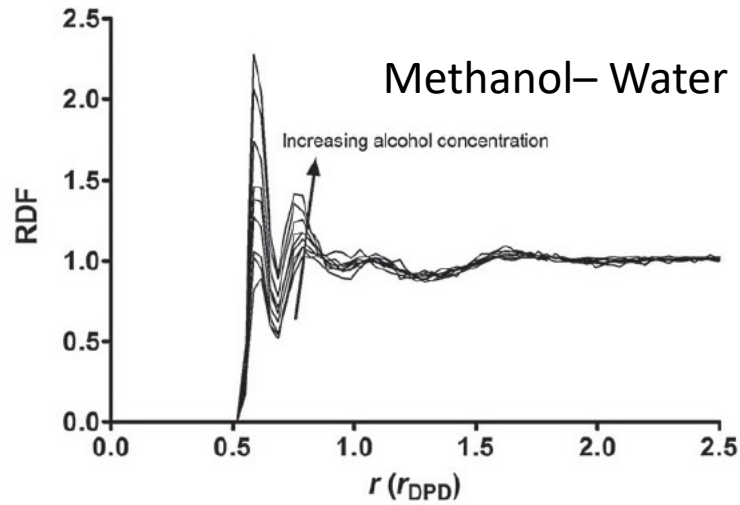
Ethanol – Water

1-Propanol – Water

Hydrogen bonding in DPD

Parameters of the Morse Potential from MD simulations

r_0 : Mapping of the Radial Distribution Functions (RDF's)



Hydrogen bonding in DPD

D_0 : Mapping of the mixing energies*

$$D_0(x) \equiv E_{\text{H-bond}}(x) = E_{\text{Pot,Mix}}(x) - E_{\text{Pot,A}} - E_{\text{Pot,W}}$$

x_{alcohol}	Methanol-Water		Ethanol-Water		1-Propanol-Water	
	D_0	r_0	D_0	r_0	D_0	r_0
	[$k_B T$]	[r_{DPD}]	[$k_B T$]	[r_{DPD}]	[$k_B T$]	[r_{DPD}]
0.10	5.06	0.78	8.22	0.78	7.21	0.60
0.20	6.33	0.78	9.14	0.78	8.28	0.69
0.30	7.46	0.78	9.97	0.78	9.23	0.73
0.39	8.68	0.78	10.85	0.78	10.25	0.75
0.51	10.07	0.78	11.86	0.78	11.42	0.77
0.59	11.12	0.78	12.62	0.78	12.3	0.77
0.69	12.41	0.78	13.56	0.78	13.38	0.78
0.84	14.20	0.78	14.86	0.78	14.88	0.78
0.93	15.31	0.78	15.66	0.78	15.82	0.78

*van der Spoel, et. al *J. Chem. Phys.* **2003**.

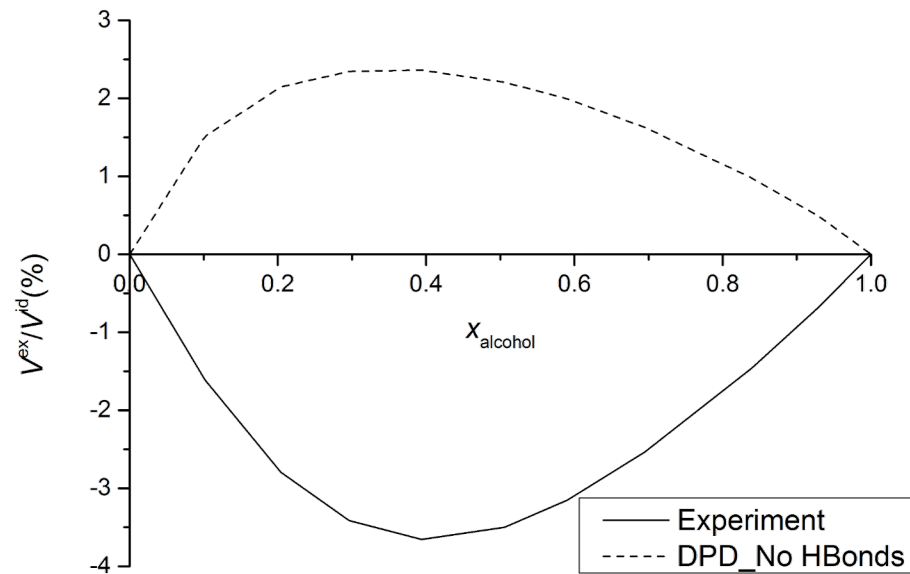
Conservative DPD interactions

Type of Bead	$\rho_{m,i}$ [g/cm ³]	M_W [g/mol]	$\rho_{i,\text{pure}} = \rho_{m,i}/M_W$ [Å ⁻³]	δ [(J/cm ³) ^{0.5}]	a_{ii} [$k_B T$]	$a_{i\text{-water}}$ [$k_B T$]
Methanol (M)	0.787	32.04	0.0148	29.61	177.44	73.27
Ethanol (E)	0.784	46.07	0.0103	26.49	409.71	107.58
1-Propanol (P)	0.799	60.09	0.0080	24.53	729.59	143.03
Water (W)	1.000	18.02	0.0334	47.00	25.00	25.00

Hydrogen bonding in DPD

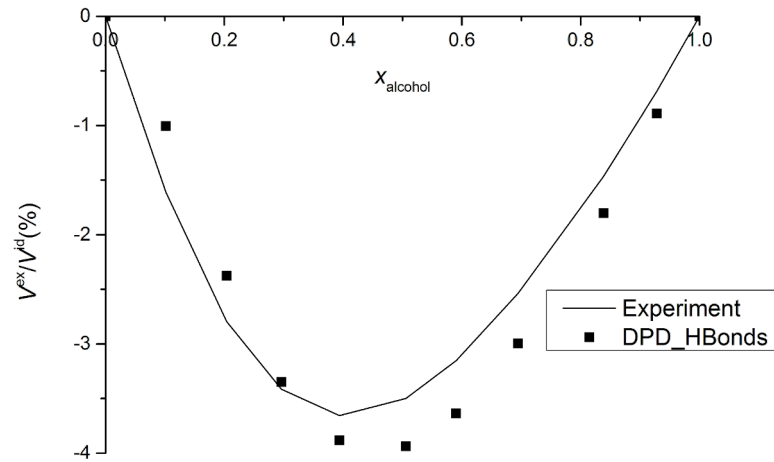
Experimental comparison of simulation results

Negative volume excess low M_W alcohol-water mixtures

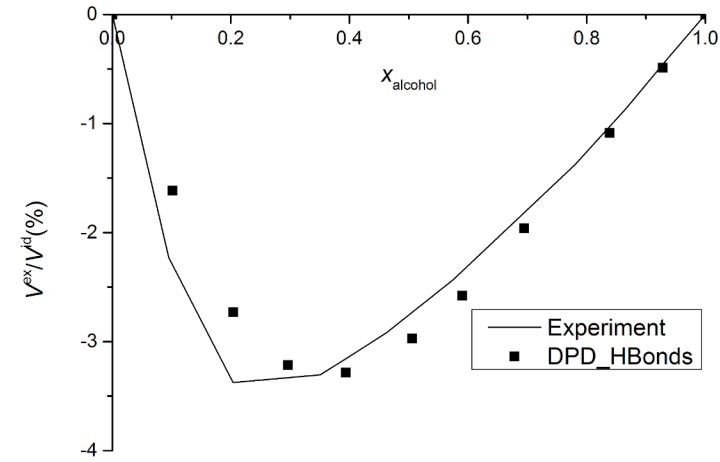


$$V^E = \frac{x_A M_A + x_B M_B}{\rho_L} - \underbrace{x_A V_{m,A} - x_B V_{m,B}}_{\text{Ideal contribution}}$$

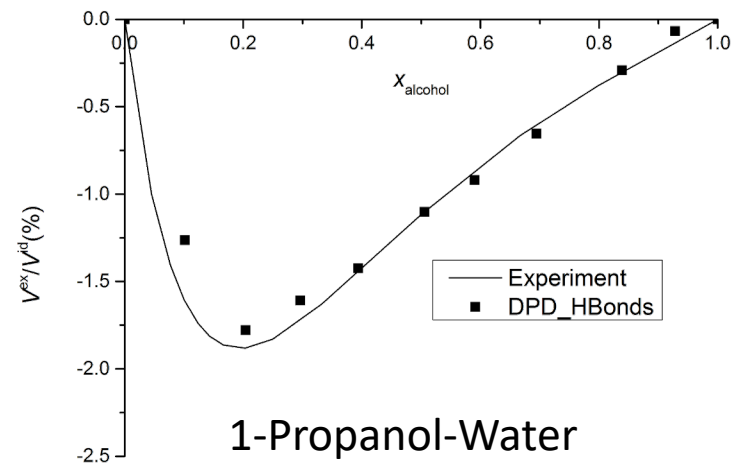
Hydrogen bonding in DPD



Methanol-Water



Ethanol-Water

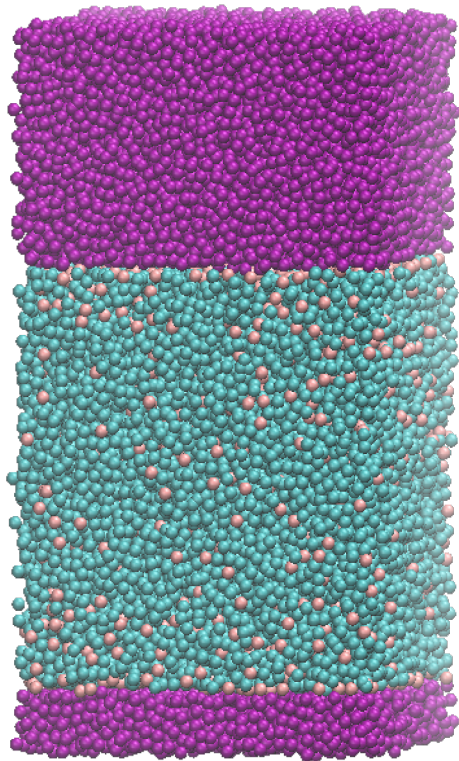


1-Propanol-Water

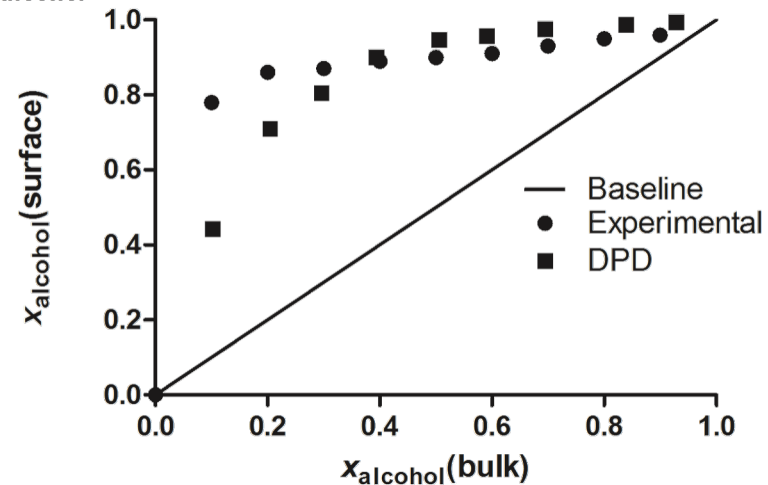
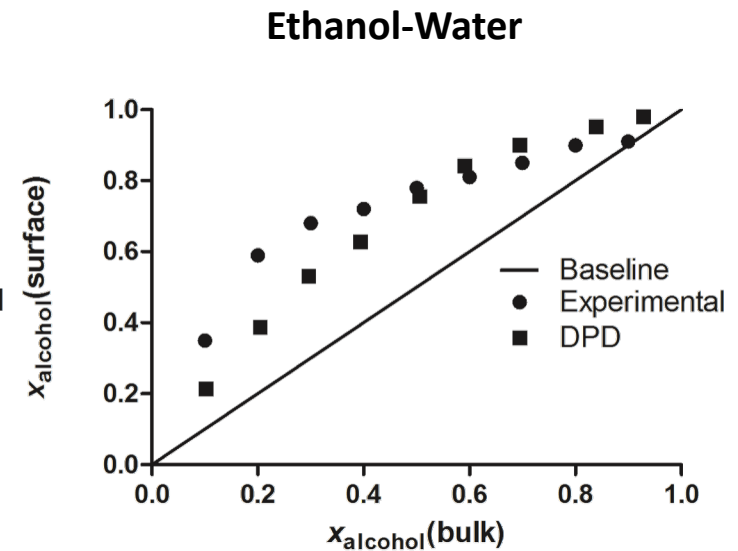
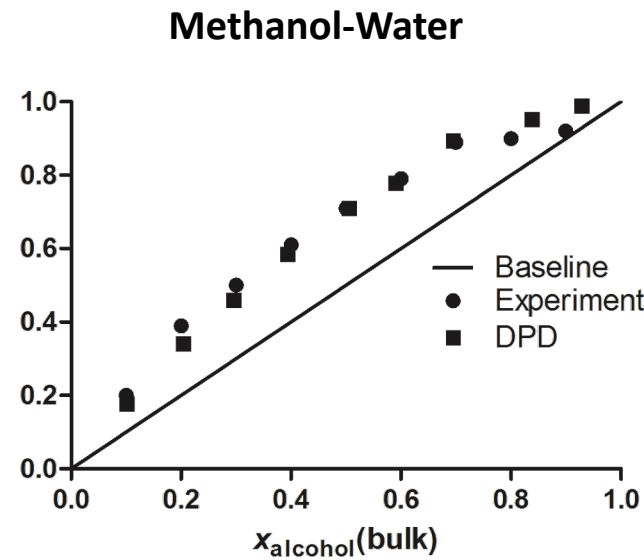
Hydrogen bonding in DPD

Experimental evidence on surface enrichment of alcohols at air interface*

*Raina et al., *J. Phys. Chem. A* 2001, 105, 10204-10207.

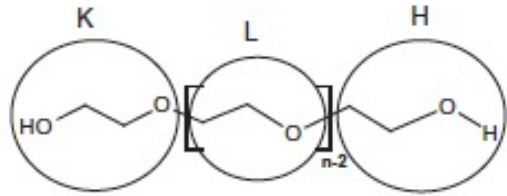


1-Propanol-Water



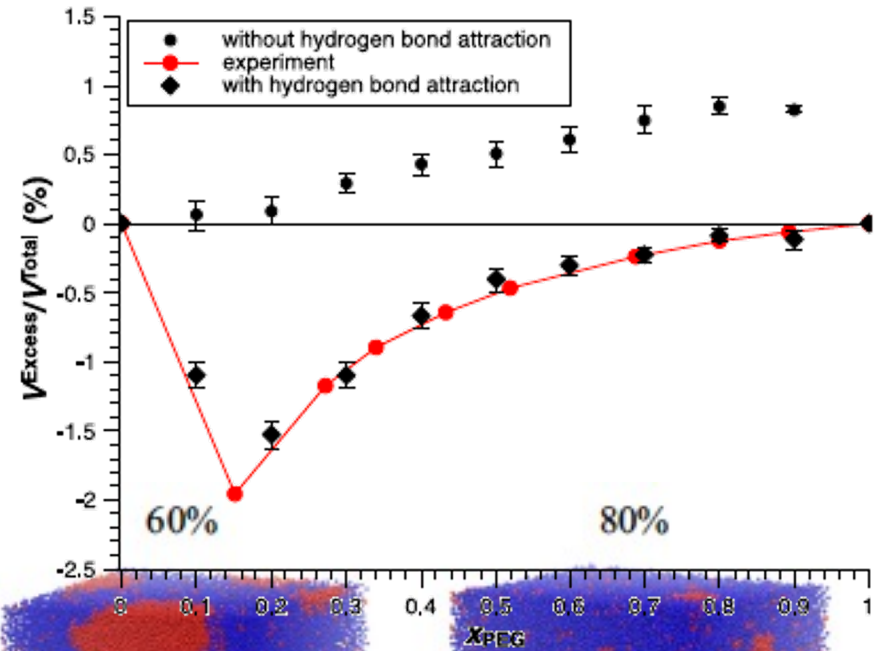
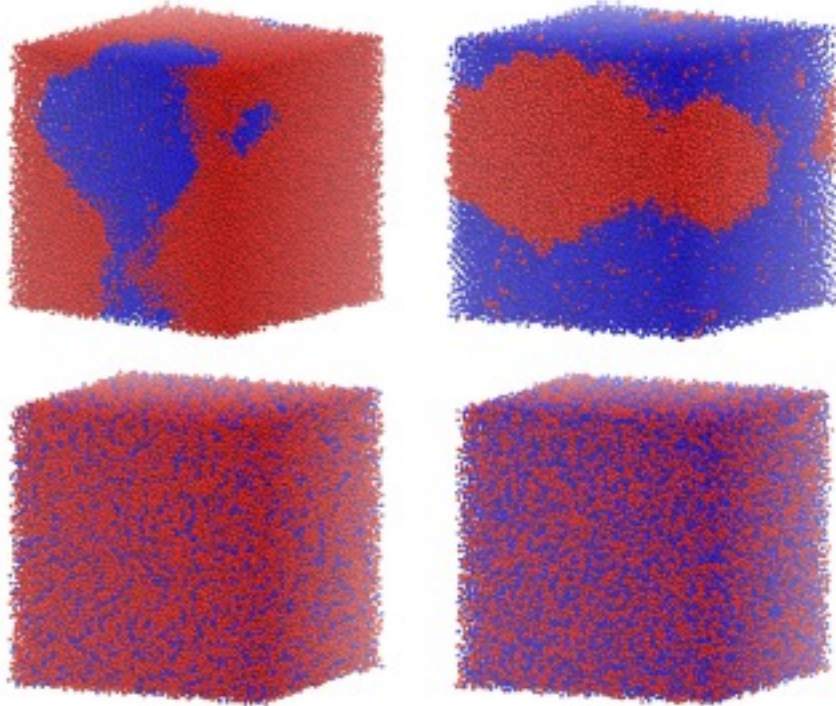
Hydrogen bonding in DPD

PEG 400 + Water interactions



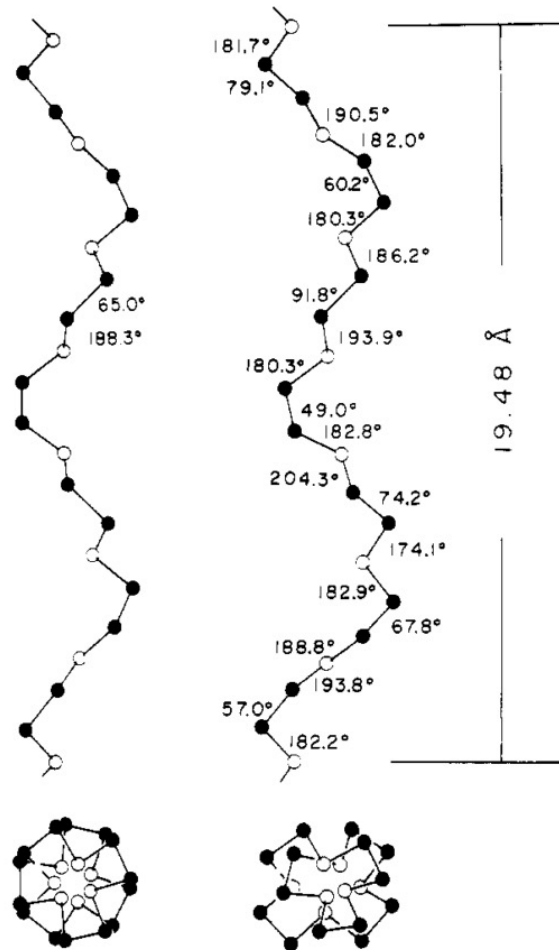
20%

40%



Single PEG chain conformations

PEG chains are known to exhibit helical structures

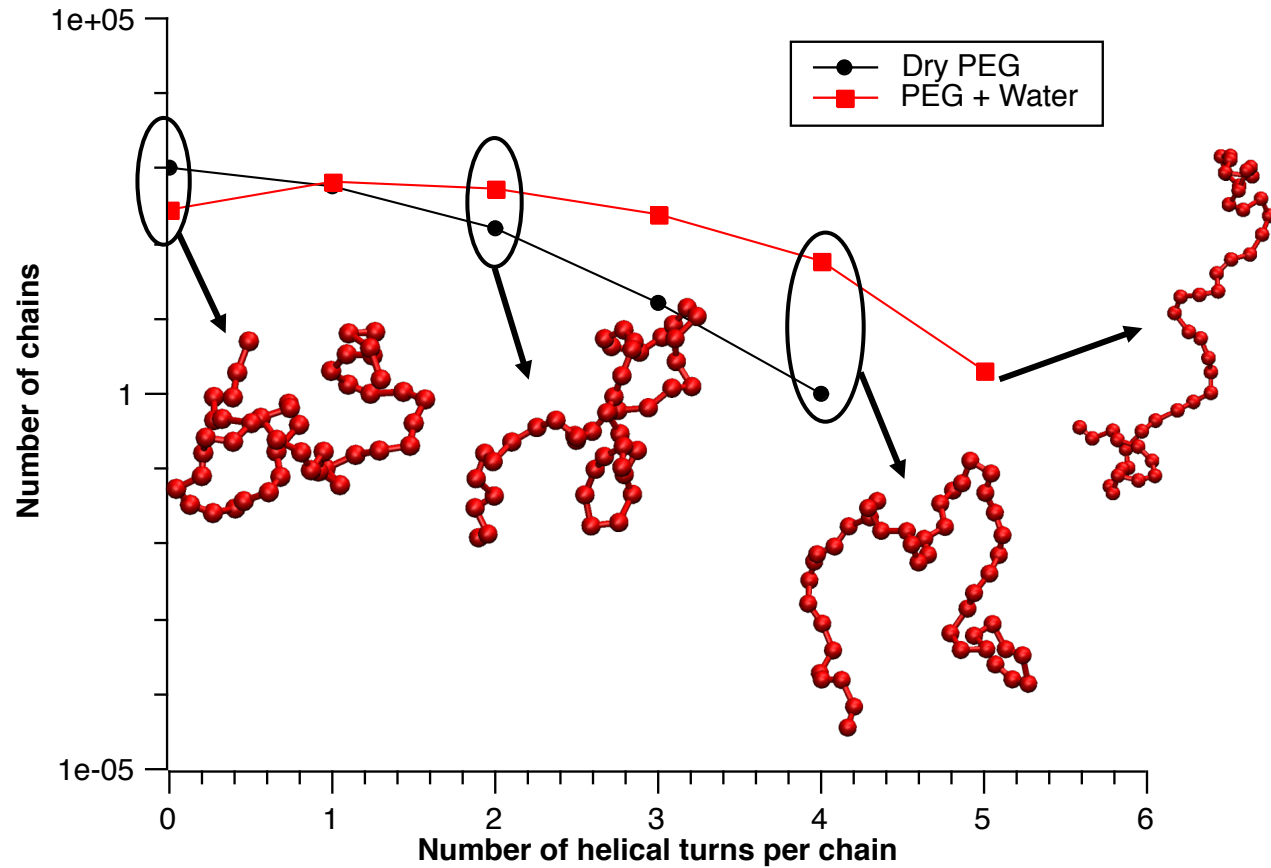


2 turns per 7 $-(\text{CH}_2-\text{CH}_2-\text{O})-$ units

Number of helical chains quantified from DPD simulations

Distance between $i-(i+3)$ and $i-(i+6)$ contacts

Single PEG chain conformations



Water significantly increases PEG chain helicity

A more generic approach

A statistical mechanics-based approach to parameterize 'hydrogen bond strength'

PHYSICAL REVIEW E **85**, 061503 (2012)

Analytical model for three-dimensional Mercedes-Benz water molecules

T. Urbic

Faculty of Chemistry and Chemical Technology, University of Ljubljana, Askerceva 5, 1000 Ljubljana, Slovenia

(Received 7 May 2012; revised manuscript received 13 June 2012; published 25 June 2012)

We developed a statistical model which describes the thermal and volumetric properties of water-like molecules. A molecule is presented as a three-dimensional sphere with four hydrogen-bonding arms. Each water molecule interacts with its neighboring waters through a van der Waals interaction and an orientation-dependent hydrogen-bonding interaction. This model, which is largely analytical, is a variant of a model developed before for a two-dimensional Mercedes-Benz model of water. We explored properties such as molar volume, density, heat capacity, thermal expansion coefficient, and isothermal compressibility as a function of temperature and pressure. We found that the volumetric and thermal properties follow the same trends with temperature as in real water and are in good general agreement with Monte Carlo simulations, including the density anomaly, the minimum in the isothermal compressibility, and the decreased number of hydrogen bonds upon increasing the temperature.

DOI: [10.1103/PhysRevE.85.061503](https://doi.org/10.1103/PhysRevE.85.061503)

PACS number(s): 61.20.Gy

Computing ϵ_{HB}

Chemical potential $\mu = -\frac{k_B T}{N} \ln Q$

$$Q = Q_1^{N/3} \quad \text{where} \quad Q_1 = (\Delta_{HB} + \Delta_{LJ} + \Delta_o)^n$$

$$\Delta_{HB} = 4\pi^2 c(T) v_{ef}^{HB} \exp\left(\frac{\epsilon_{HB} + \epsilon_{LJ} - P v_{HB}/2}{k_B T}\right) \sqrt{\frac{4\pi k_B T}{k_s}} \operatorname{erf}\left(\sqrt{\frac{k_s}{4k_B T}}\right)$$

$$\Delta_{LJ} = 2\pi^2 c(T) v_{ef}^{LJ} \exp\left(\frac{\epsilon_{LJ} - P v_{LJ}/2}{k_B T}\right)$$

$$\Delta_o = 2\pi^2 c(T) \frac{k_B T}{p} \exp\left(\frac{-P v_o}{2k_B T}\right)$$

$$\mu(\text{exp}) = -237.14 \frac{\text{kJ}}{\text{mol}} \rightarrow \epsilon_{HB} = 14.6 k_B T$$

DPD modeling of simple hydrogen bonding liquids

Water, ethanol, methanol, 1-propanol

Hydrophobic and hydrophilic parts form different CG beads

CG beads are connected *via* covalent bonds

Only h-bonds between hydrophilic beads are defined

Properties estimated

- RDFs

- 3-body angular distributions

- Transport properties

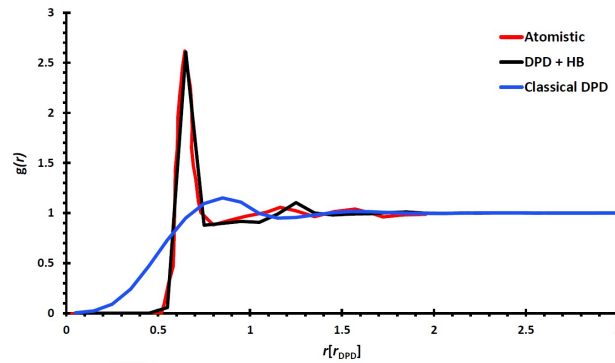
- Thermodynamic properties

- Temperature-dependent properties

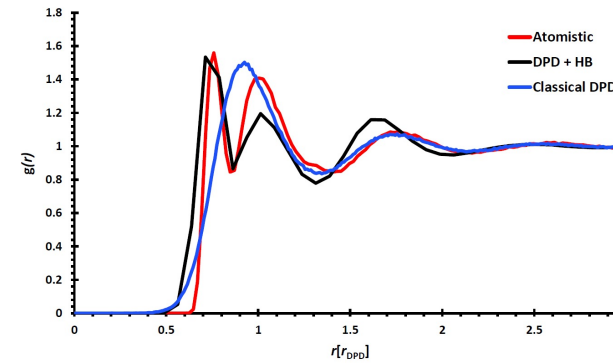
RDFs of pure liquids

RDFs of pure liquids

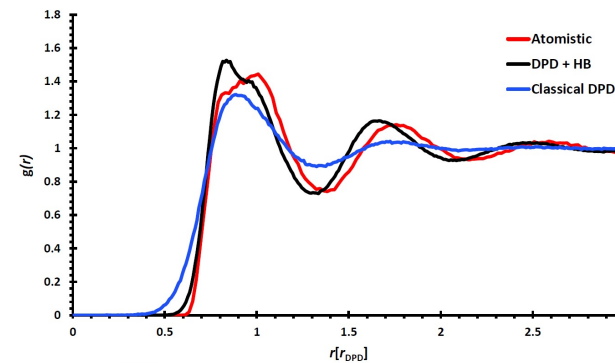
$$g_{ij}(r) = \frac{\langle \Delta N_{ij}(r \rightarrow r + \Delta r) \rangle V}{4\pi r^2 \Delta r N_i N_j}$$



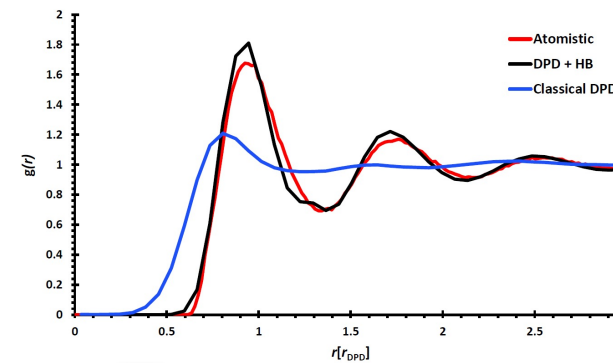
Water



Methanol

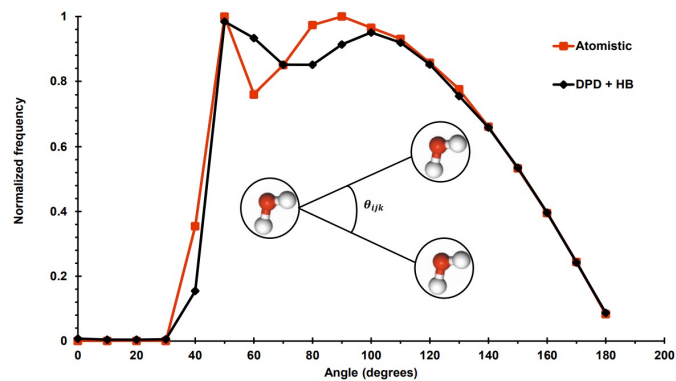


Ethanol

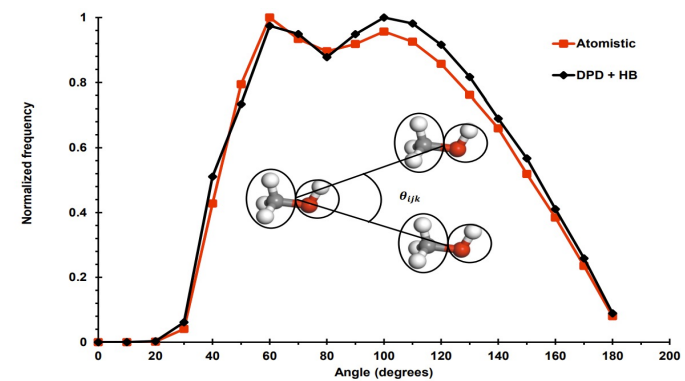


1-propanol

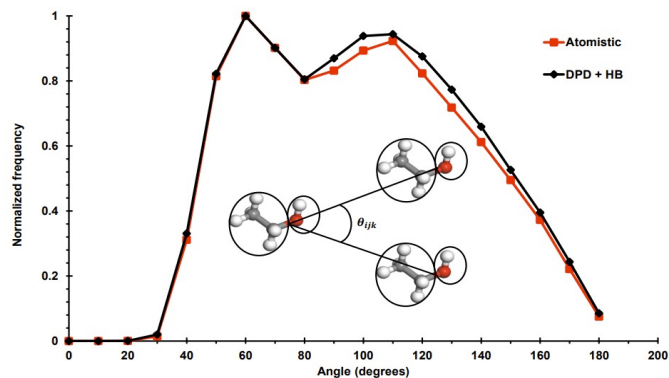
3-body angular distributions



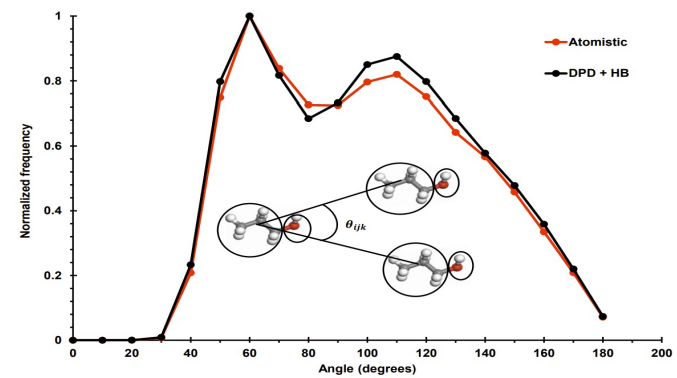
Water



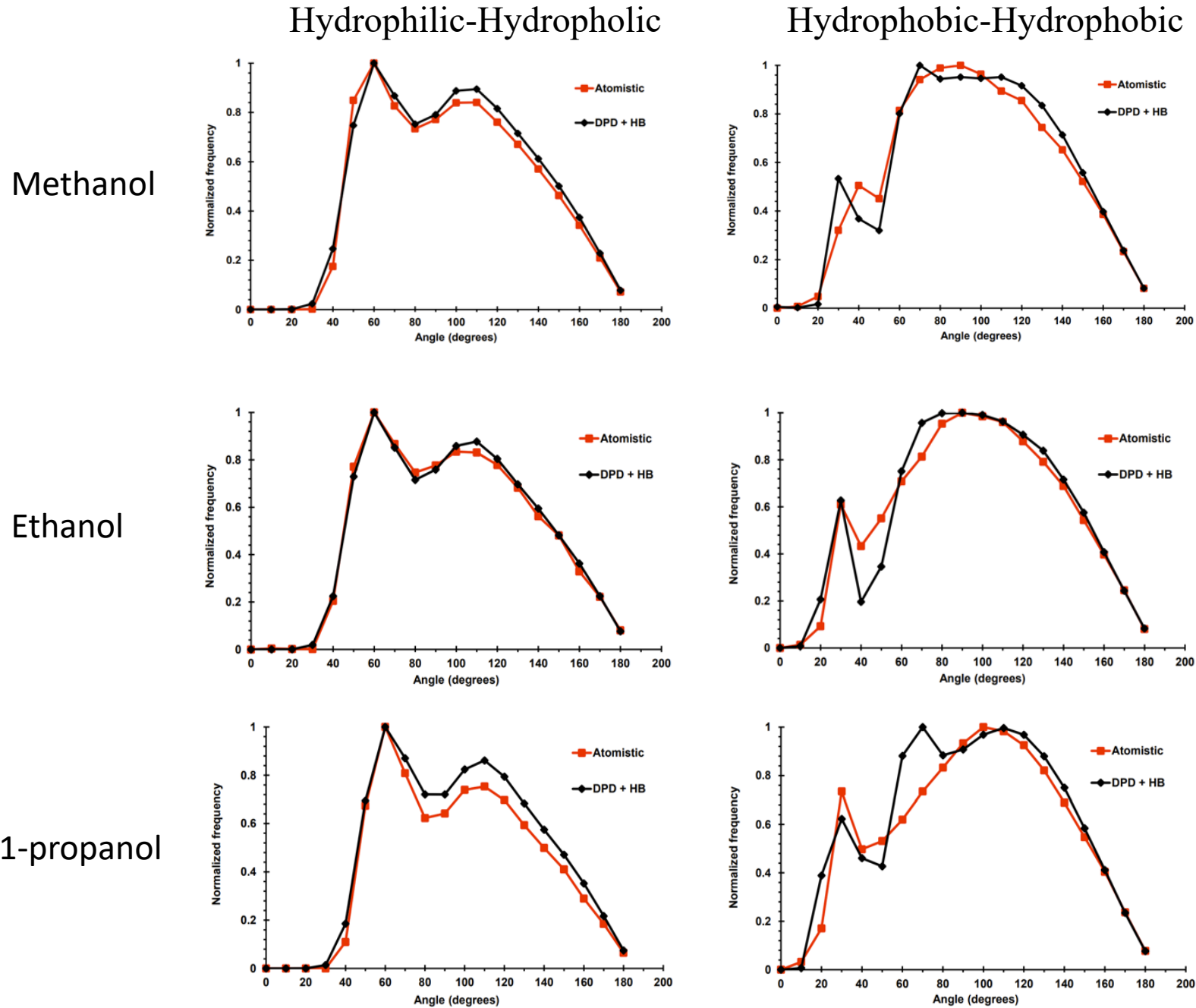
Methanol



Ethanol



1-propanol



Transport properties: Viscosity

Green-Kubo formalism

Shear viscosity

$$\eta = \frac{V}{k_B T} \int_0^\infty dt \langle P_{\text{cross}}(0) P_{\text{cross}}(t) \rangle \text{ where } P_{\text{cross}} = (P_{xy} + P_{yz} + P_{xz})/3$$

Mapping diffusion equations from DPD to experimental values

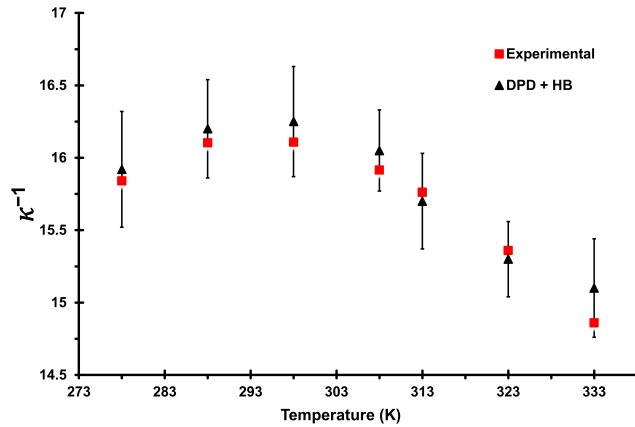
$$\tau = \frac{N_m D_{\text{DPD}} r_{\text{DPD}}^2}{D_{\text{exp}}}$$

$$\eta_{\text{real}} = \eta_{\text{DPD}} \tau k_B T / r_{\text{DPD}}^3$$

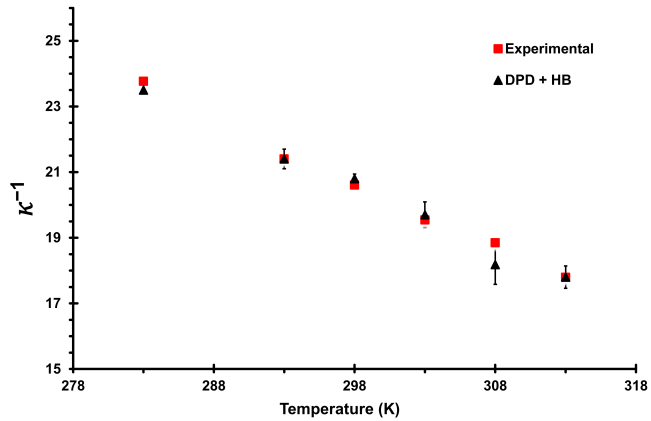
	D_{DPD} [$r_{\text{DPD}}^2/t_{\text{DPD}}$]	D_{exp} $\times 10^{-9}$ [m ² /s]	τ [ps]	η_{DPD} [$k_B T / r_{\text{DPD}}^3 \cdot \text{s}$]	η_{real} $\times 10^{-3}$ [Pa.s]	η_{exp} $\times 10^{-3}$ [Pa.s]
Water	7.27×10^{-2}	2.42	6.02	3.20	0.88	0.89
Methanol	2.32×10^{-2}	2.27	2.06	5.65	0.53	0.54
Ethanol	1.80×10^{-2}	1.01	4.61	6.68	0.97	1.00
1-propanol	6.17×10^{-3}	0.59	3.19	15.45	1.21	1.95

Thermodynamic properties: Isothermal compressibility

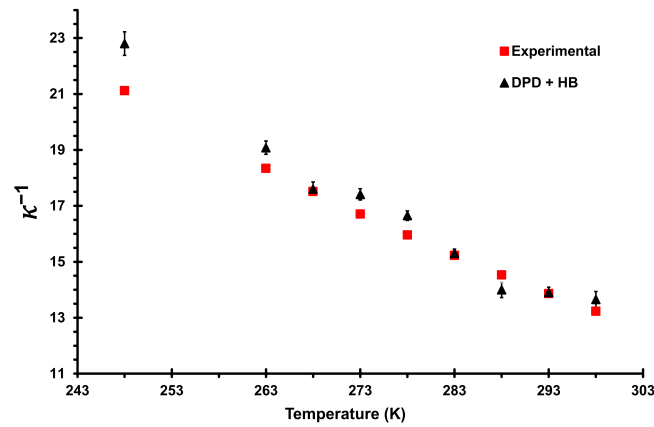
$$\kappa_T = \frac{1}{k_B T} \frac{\langle V^2 \rangle - \langle V \rangle^2}{\langle V \rangle} \quad \text{and} \quad \kappa^{-1} = \frac{1}{\rho k_B T \kappa_T}$$



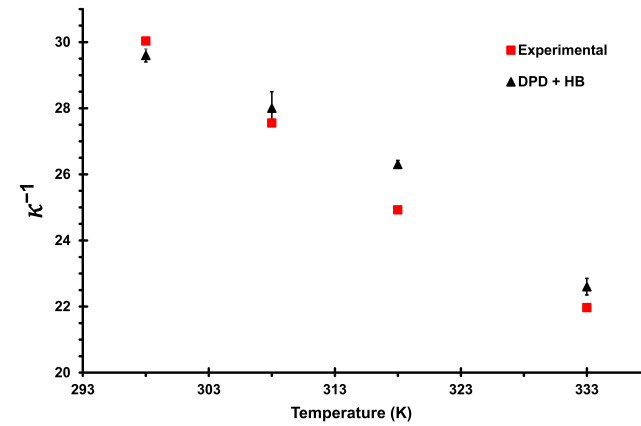
Water



Ethanol



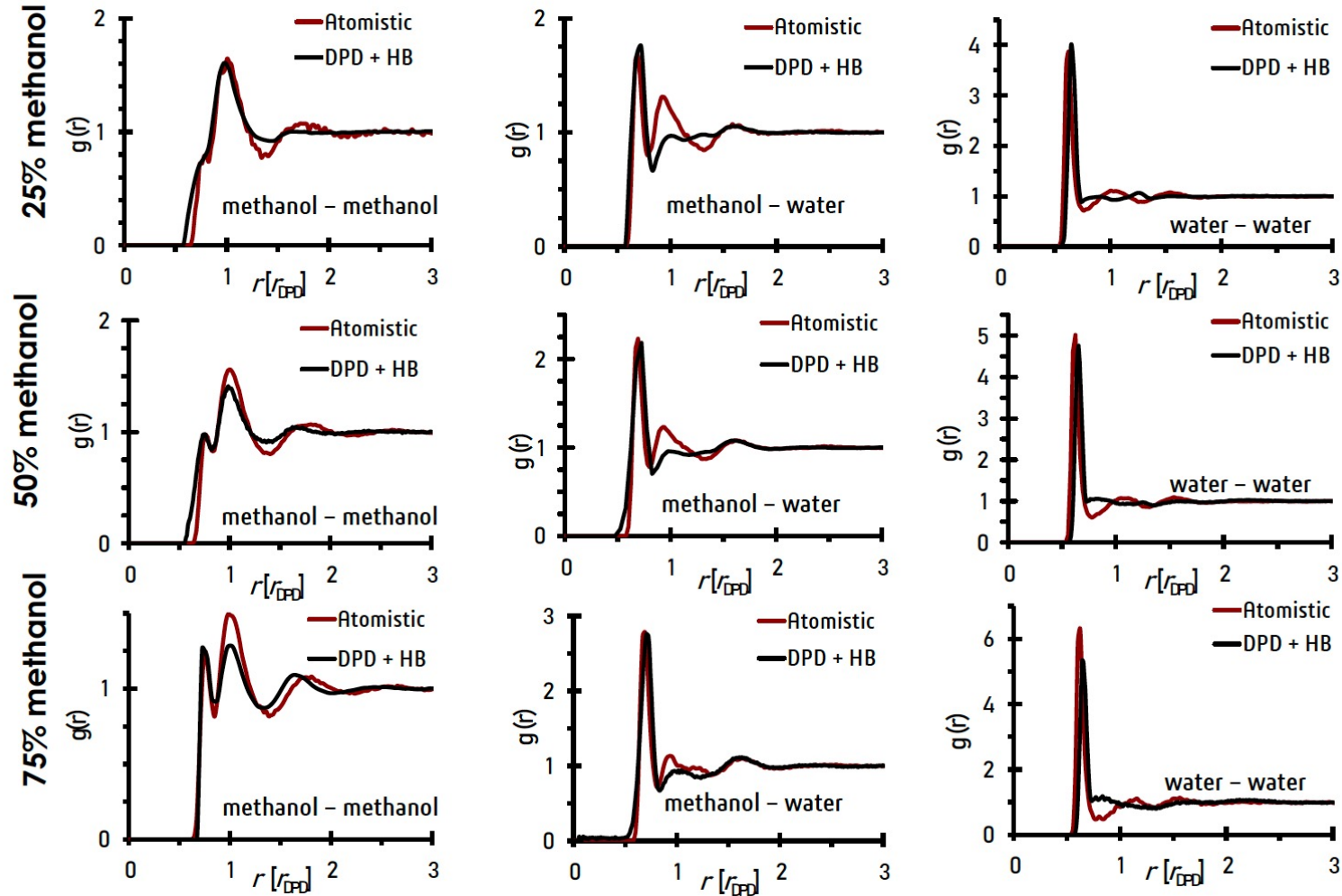
Methanol



1-propanol

Mixtures

Mixtures of simple liquids: Case of Methanol

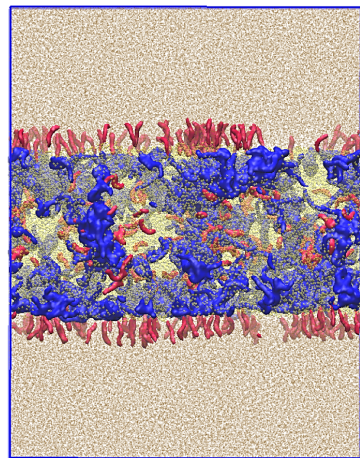


Simulated soft matter by DPD

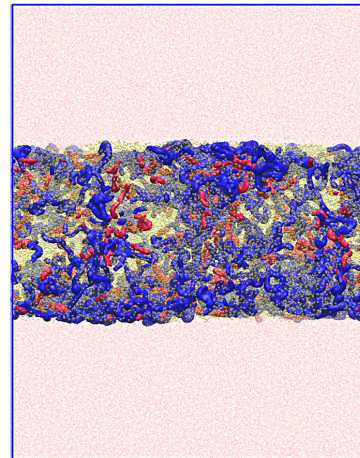
Surface segregation of PU coatings

Adaptive surface behavior

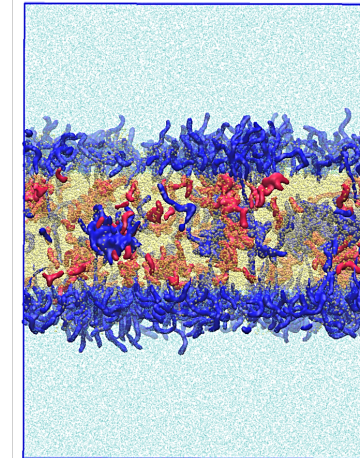
Oil



Air



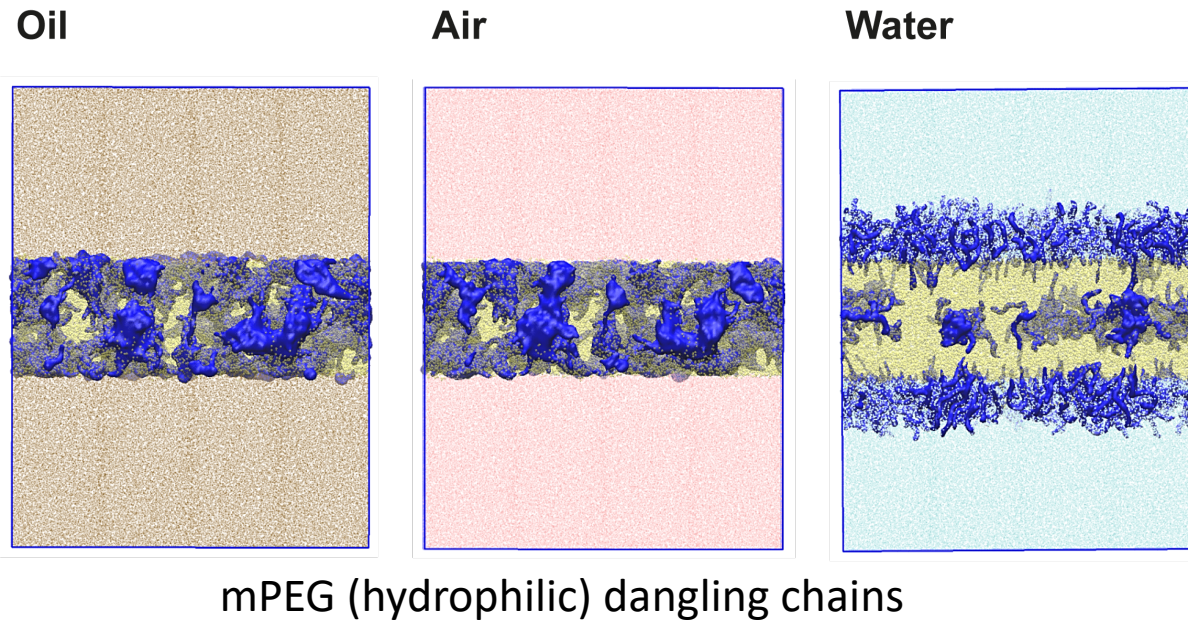
Water



mPEG (hydrophilic) and oDEC (hydrophobic) dangling chains

Surface segregation of PU coatings

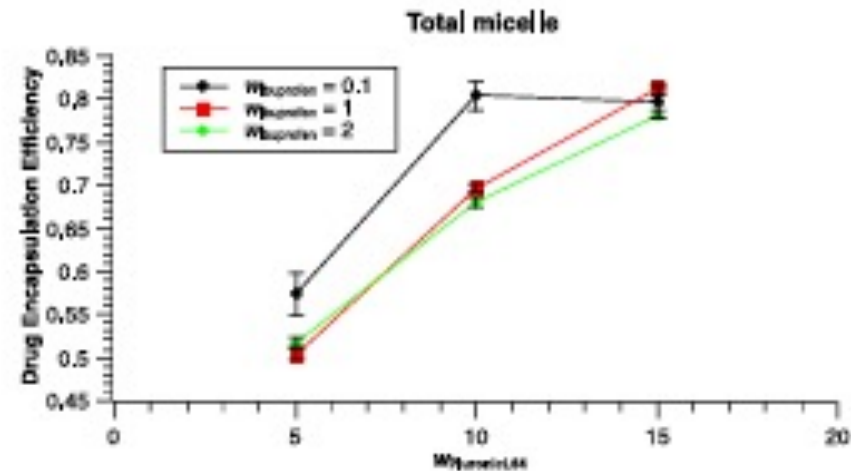
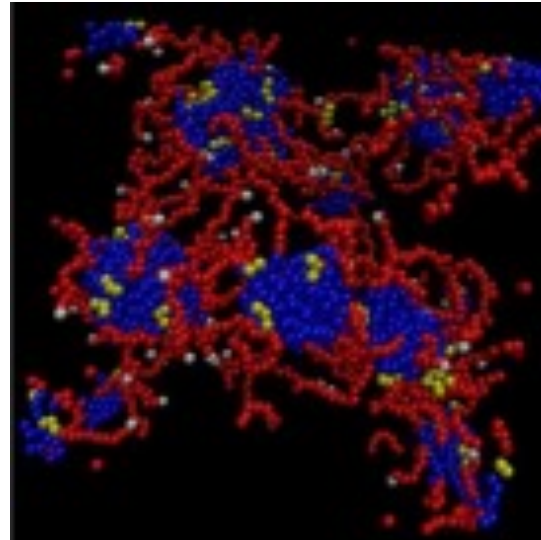
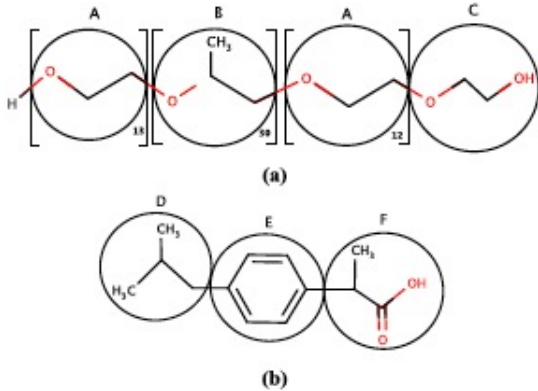
Adaptive surface behavior



Polymeric drug delivery systems

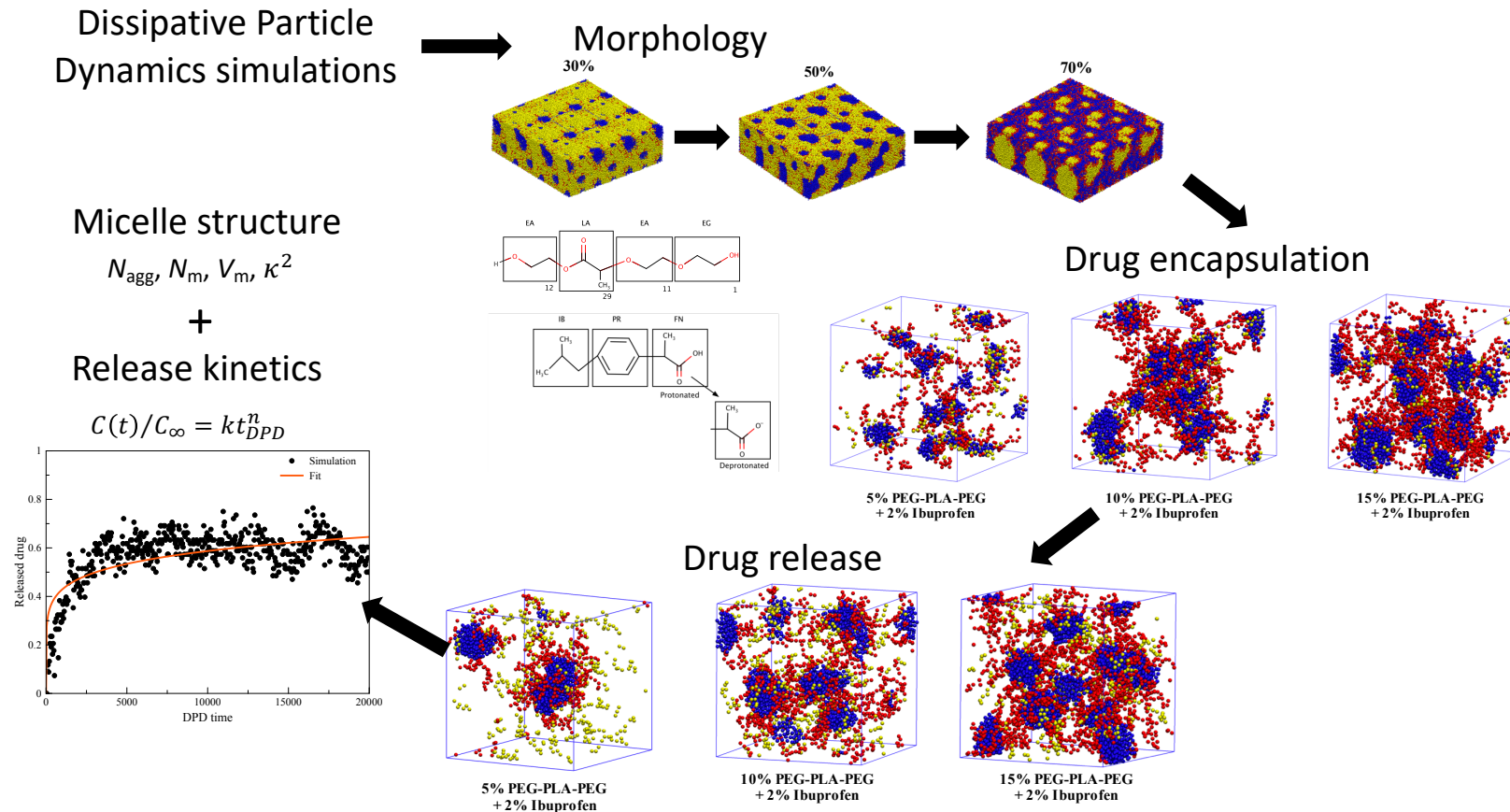
PEG-PPO-PEG micelles

Ibuprofen



Polymeric drug delivery systems

PEG-PLA-PEG micelles



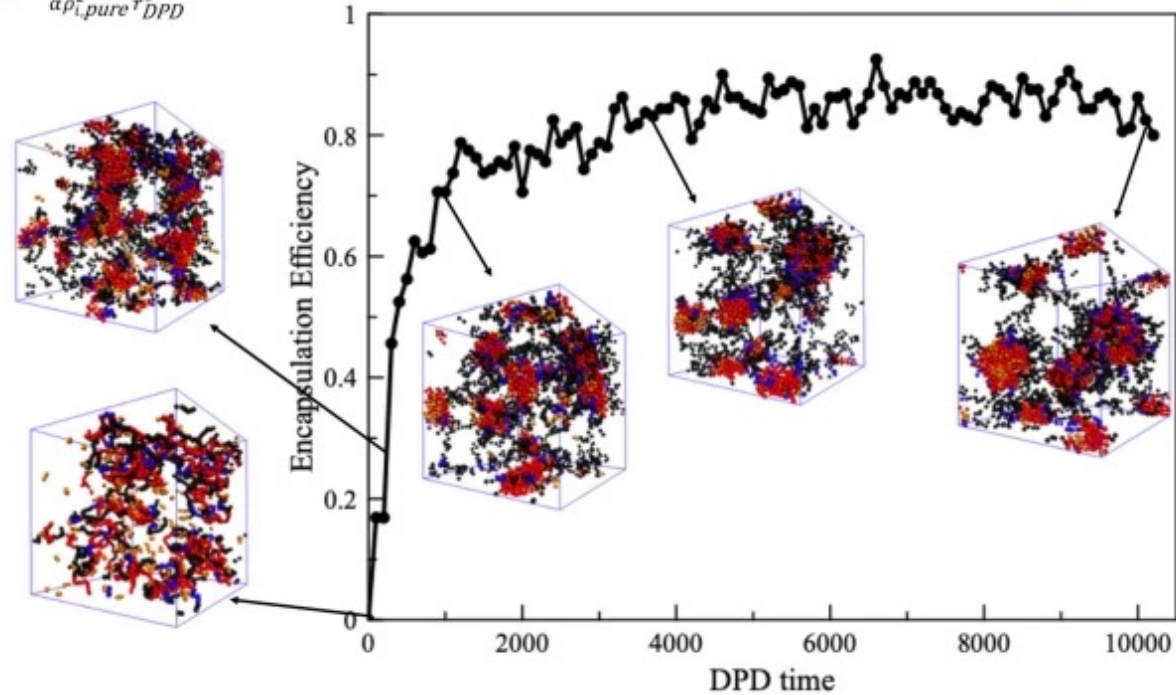
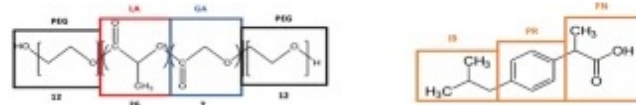
Polymeric drug delivery systems

PEG-PLGA-PEG micelles

Dissipative Particle Dynamics simulations

$$a_{ij} = \tilde{a}_{ij} + \frac{p}{0.0454(a_{ii}\rho_{i,pure} + a_{jj}\rho_{j,pure})} \chi_{ij} k_B T$$

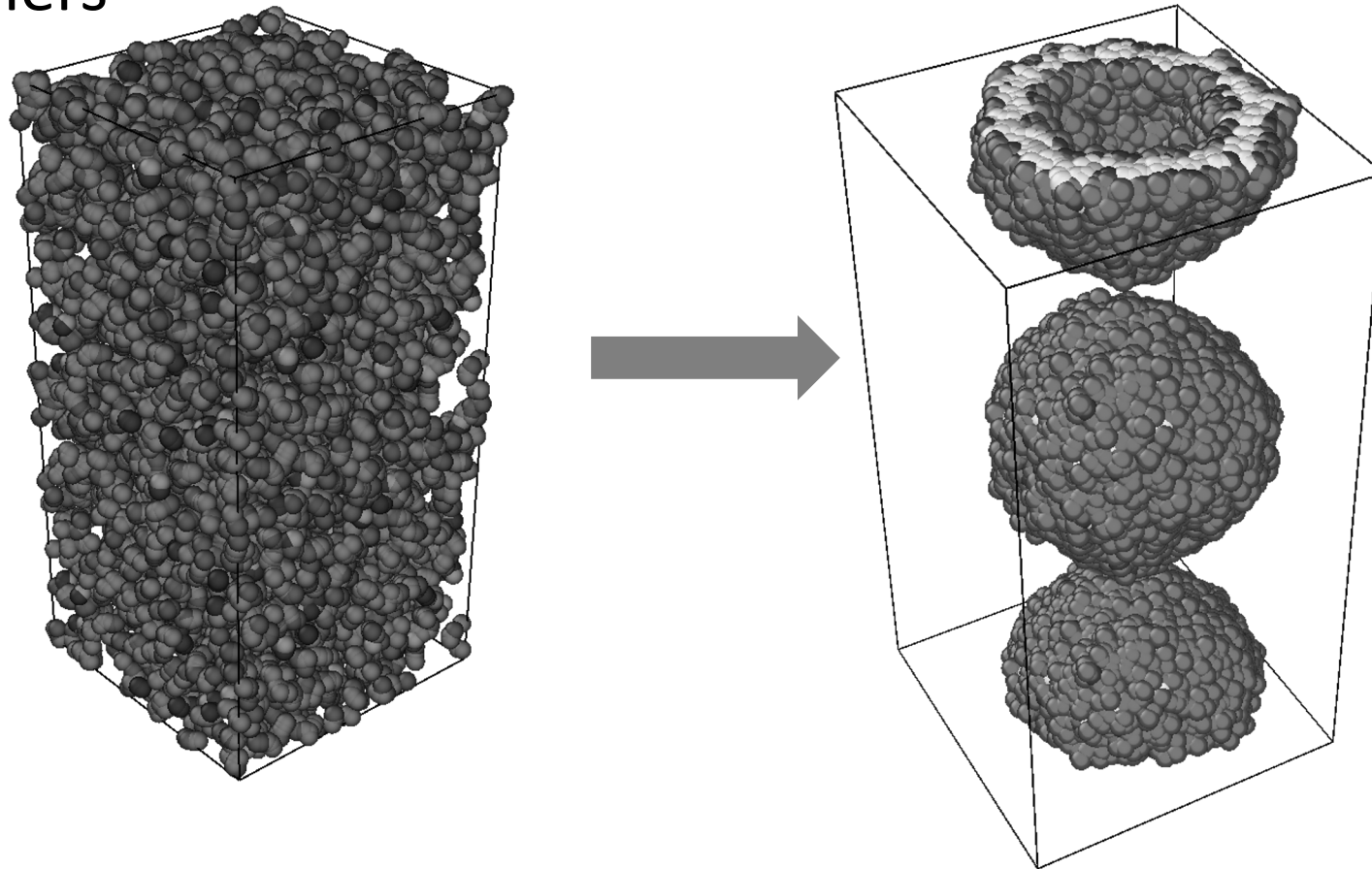
$$a_{ii} = \frac{p - \rho_{i,pure} k_B T}{\alpha \rho_{i,pure}^2 DPD}$$

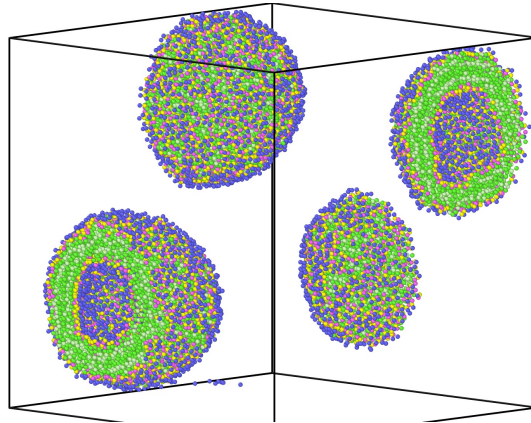
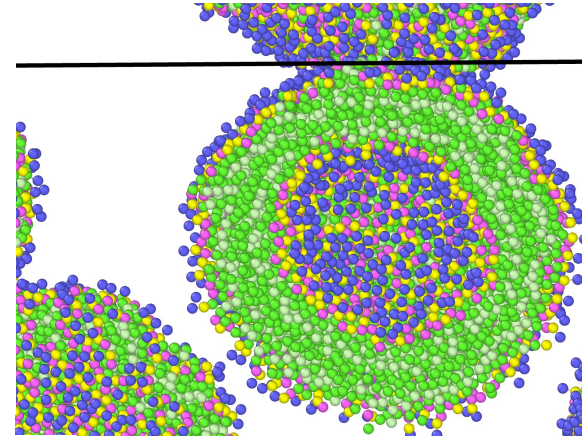
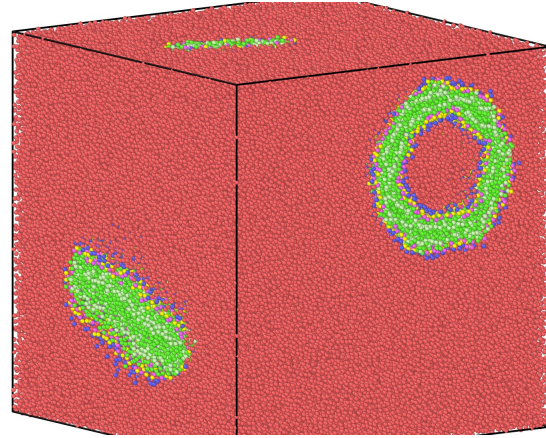
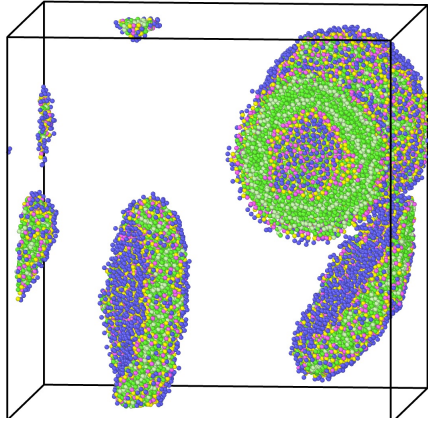
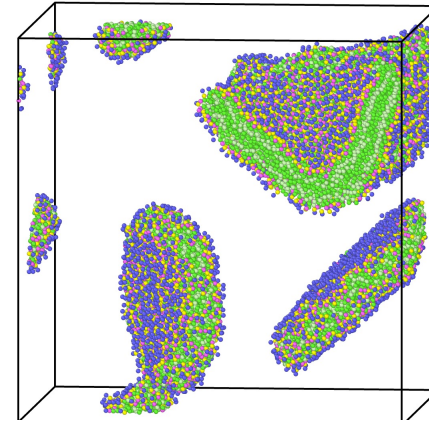
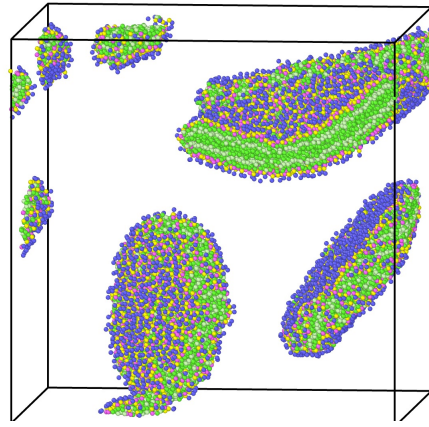
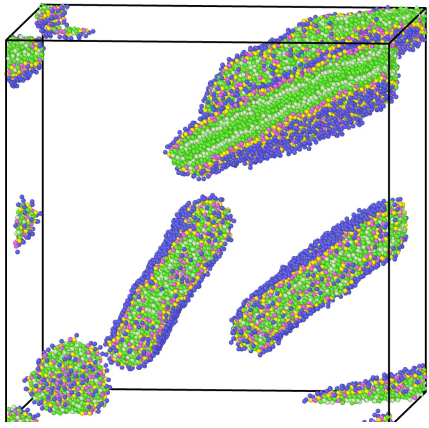


Lipid-based carriers

Coenzyme Q10 encapsulation in lecithin liposomes

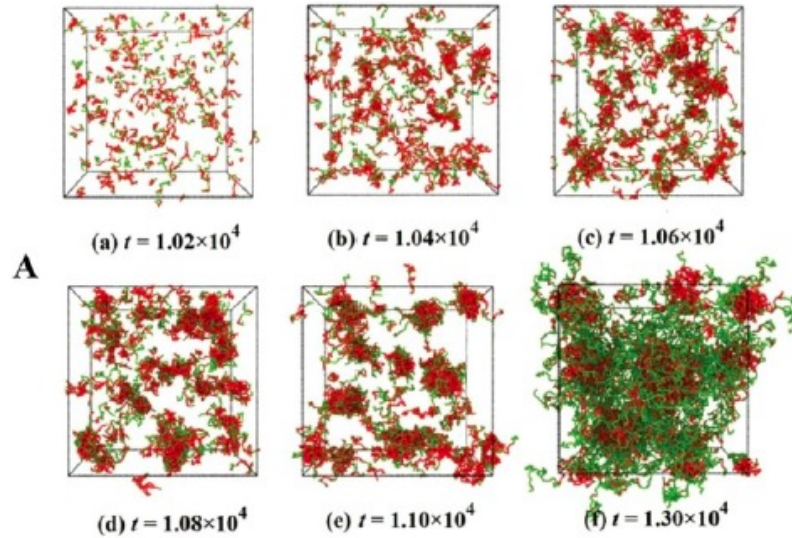
mRNA carriers





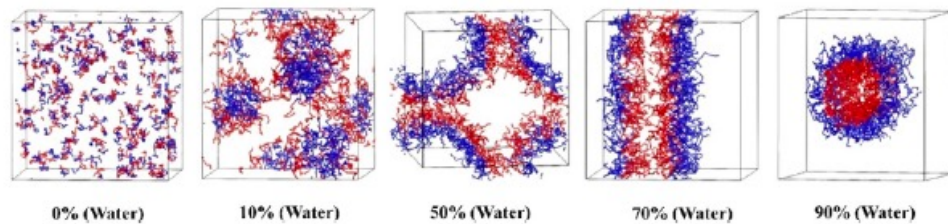
Other DPD examples

Polymer micelle self-assembly



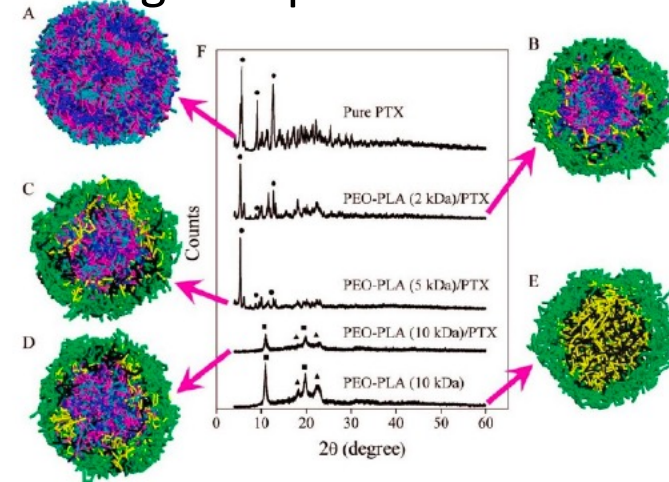
Kong, S. M.; *v.d.* Phys. Chem. Chem. Phys., 2018.

Micellization behavior



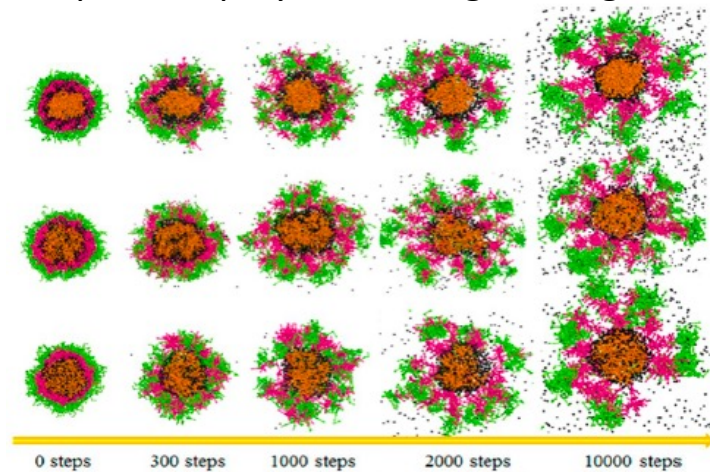
Huo, J. H.; Chen, Z.; Zhou, J. Langmuir, 2019.

Drug encapsulation and XRD



Guo, X. D.; Zhang, L. J.; Qian, Y. Ind. Eng. Chem. Res., 2012.

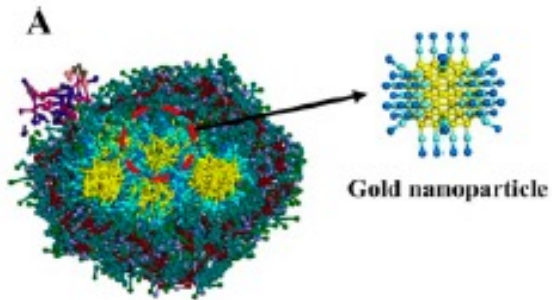
pH responsive polymer – drug loading/release



Nie, S. Y.; *v.d.* ACS Appl. Mater. Interfaces, 2014.

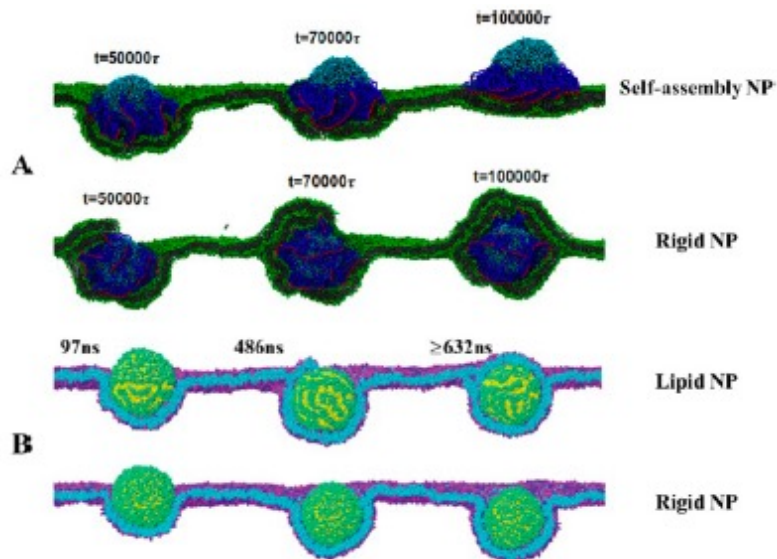
Other DPD examples

siRNA – polymer – Au NP



Xie, X. N. *v.d. AIChE J.*, 2018.

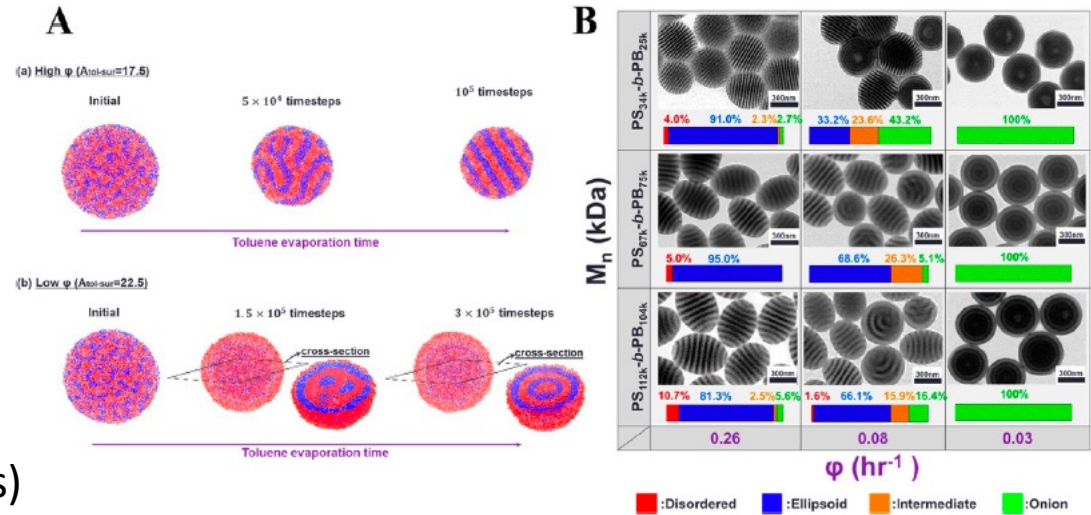
RME (Receptor mediated endocytosis)



Shen, Z. Q.; Ye, H. L.; Kroger, M.; Li, Y. *Nanoscale*, 2018.

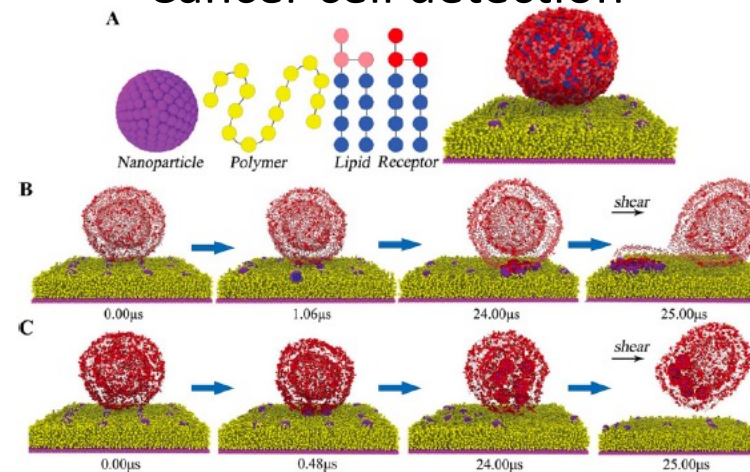
Bai, X.; Xu, M.; Liu, S. J.; Hu, G. Q. *ACS Appl. Mater. Interfaces*, 2018.

DPD – TEM comparison



Shin, J. M.; Kim, Y.; Yun, H.; Yi, G. R.; Kim, B. J., *ACS Nano*, 2017.

Cancer cell detection



Huang, L. Y.; Yu, Y. S.; Lu, X.; Ding, H. M.; Ma, Y. Q. *Nanoscale*, 2019.

Hands-on session

1. Phase separation of monomers

Monitor phase separation process

Observe repulsion strength dependency on phase separation

- 1:1 A:B mixture

2. Micelle formation

Monitoring micelle formation process

Effect of concentration

- A_x - B_x type diblock copolymers in solvent

3. Cross-linking

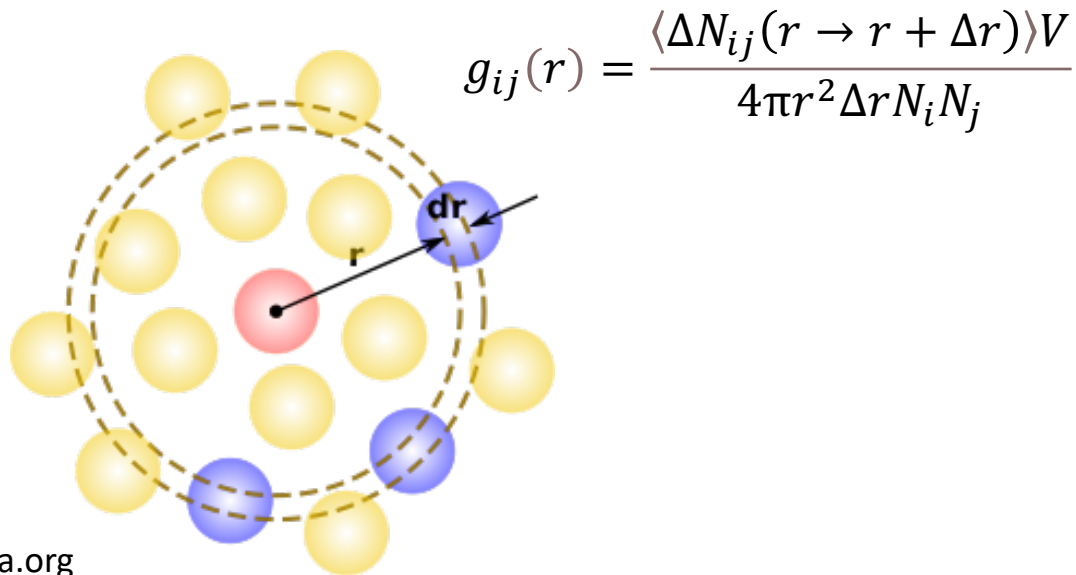
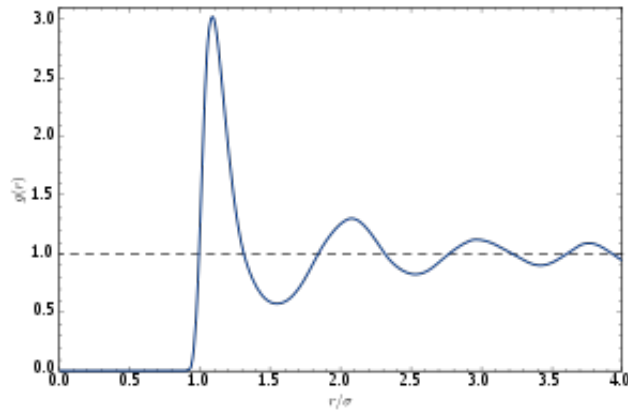
Modeling chemical reactions in DPD

A-B-B-A oligomer + star-shaped cross-linker

Dynamics of cross-link formation

Properties

Radial Distribution Function



Isothermal compressibility

$$\kappa_T = \frac{1}{k_B T} \frac{\langle V^2 \rangle - \langle V \rangle^2}{\langle V \rangle} \quad \text{and} \quad \kappa^{-1} = \frac{1}{\rho k_B T \kappa_T}$$

Mean Squared Displacement

$$\text{MSD} = \frac{1}{N} \sum_{i=1}^N |r^i(t) - r^i(0)|^2$$

Cross-link conversion

$$X = \frac{N_{\text{new formed}}}{N_{\text{Total}}}$$

Density profile

$$\rho = \frac{N_{\text{beads}}}{V_{\text{Slab}}}$$

Time for fun!

